

# Package ‘MN’

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**Type** Package

**Title** Matrix Normal Distribution

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**Imports** rangen, Rfast

**Description** Density computation, random matrix generation, maximum likelihood estimation, and regression for the matrix normal distribution. References: Pocuca N., Gal-  
laugher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Vari-  
ate Normality. <[doi:10.48550/arXiv.1910.02859](https://doi.org/10.48550/arXiv.1910.02859)> and the relevant wikipedia page.

**License** GPL (>= 2)

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 MN-package

*Matrix Normal Distribution*


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### Description

Density computation, random matrix generation and maximum likelihood estimation of the matrix normal distribution. For references see: Pocuca N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859 and the relevant wikipedia page.

### Details

Package: MN  
 Type: Package  
 Version: 1.2  
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### Maintainers

Michail Tsagris <mtsagris@uoc.gr>.

### Author(s)

Michail Tsagris <mtsagris@uoc.gr> and Omar Alzeley <oazeley@uqu.edu.sa>

### References

Pocuca, N., Gallagher, M. P., Clark, K. M., & McNicholas, P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

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 Density of the matrix normal distribution

*Density of the matrix normal distribution*


---

### Description

Density of the matrix normal distribution.

### Usage

```
dmn(X, M, U, V, logged = FALSE)
```

**Arguments**

|                     |                                                                                                                                                                                 |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>X</code>      | A list with $k$ elements, $k$ matrices of dimension $n \times p$ each. In the case of one matrix only, this may be given as a numerical matrix and not as an element in a list. |
| <code>M</code>      | The mean matrix of the distribution, a numerical matrix of dimensions $n \times p$ .                                                                                            |
| <code>U</code>      | The covariance matrix associated with the rows, a numerical matrix of dimensions $n \times n$ .                                                                                 |
| <code>V</code>      | The covariance matrix associated with the columns, a numerical matrix of dimensions $p \times p$ .                                                                              |
| <code>logged</code> | Should the logarithm of the density be computed?                                                                                                                                |

**Value**

A numeric vector with the (logged) density values.

**Author(s)**

Omar Alzeley.

R implementation and documentation: Omar Alzeley <oazeley@uqu.edu.sa>.

**References**

[https://en.wikipedia.org/wiki/Matrix\\_normal\\_distribution#Definition](https://en.wikipedia.org/wiki/Matrix_normal_distribution#Definition)

Pocuca, N., Gallagher, M. P., Clark, K. M., & McNicholas, P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

**See Also**

[rmn](#), [mn.mle](#), [ddplot](#)

**Examples**

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
X <- rmn(10, M, U, V)
dmn(X, M, U, V, TRUE)
```

---

Distance-Distance Plot

*Distance-Distance Plot*


---

**Description**

Distance-Distance Plot

**Usage**

```
ddplot(X, M, U, V)
```

**Arguments**

|   |                                                                                                                                                                             |
|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| X | A list with k elements, k matrices of dimension $n \times p$ each. In the case of one matrix only, this may be given as a numerical matrix and not as an element in a list. |
| M | The mean matrix of the distribution, a numerical matrix of dimensions $n \times p$ .                                                                                        |
| U | The covariance matrix associated with the rows, a numerical matrix of dimensions $n \times n$ .                                                                             |
| V | The covariance matrix associated with the columns, a numerical matrix of dimensions $p \times p$ .                                                                          |

**Details**

The distance-distance plot is produced. This is a scatter plot of the Mahalanobis distances computed using the estimated parameters from the multivariate normal and matrix normal distribution. See Pocuca et al. (2019) for more details.

**Value**

A scatter plot of the Mahalanobis distances.

**Author(s)**

Michail Tsagris and Omar Alzeley.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr> and Omar Alzeley <oazeley@uqu.edu.sa>.

**References**

Pocuca N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

**See Also**

[rmn](#), [mn.mle](#), [dmn](#), [ddktest](#)

**Examples**

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
X <- rmn(100, M, U, V)
ddplot(X, M, U, V)
```

---

Kolmogorov-Smirnov test for matrix normality

*Kolmogorov-Smirnov test for matrix normality*


---

**Description**

Kolmogorov-Smirnov test for matrix normality

**Usage**

```
ddkstest(X, M, U, V, alpha = 0.05)
```

**Arguments**

|       |                                                                                                                                                                             |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| X     | A list with k elements, k matrices of dimension $n \times p$ each. In the case of one matrix only, this may be given as a numerical matrix and not as an element in a list. |
| M     | The mean matrix of the distribution, a numerical matrix of dimensions $n \times p$ .                                                                                        |
| U     | The covariance matrix associated with the rows, a numerical matrix of dimensions $n \times n$ .                                                                             |
| V     | The covariance matrix associated with the columns, a numerical matrix of dimensions $p \times p$ .                                                                          |
| alpha | The significance level for the test, set by default equal to 0.05.                                                                                                          |

**Details**

The Kolmogorov-Smirnov test for matrix normality is performed. See Pocuca (2019) for more details.

**Value**

A message. If the Kronecker product covariance structure is not present, the message reads "Reject" and "Not reject otherwise".

**Author(s)**

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

## References

Pocuca N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

## See Also

[rmn](#), [mn.mle](#), [dmn](#), [ddplot](#)

## Examples

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
X <- rmn(200, M, U, V)
ddkstest(X, M, U, V)
```

---

Log-likelihood ratio test for equality of all parameters

*Log-likelihood ratio test for equality of all parameters*

---

## Description

Log-likelihood ratio test for equality of all parameters.

## Usage

```
mn.equaltest(Y1, Y2, tol = 1e-6)
```

## Arguments

|     |                                                                                                                                              |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------|
| Y1  | The observations of the first sample. A list with $k_1$ elements ( $k_1$ is the sample size), $k_1$ matrices of dimension $n \times p$ each. |
| Y2  | The observations of the first sample. A list with $k_2$ elements ( $k_2$ is the sample size), $k_2$ matrices of dimension $n \times p$ each. |
| tol | The tolerance value to terminate the iterative algorithm.                                                                                    |

## Details

A log-likelihood ratio test comparing the equality of all parameters, mean matrices and covariance matrices mean matrices is performed.

**Value**

This is an "htest" class object. Thus it returns a list including:

|             |                                              |
|-------------|----------------------------------------------|
| statistic   | The test statistic value.                    |
| parameter   | The degrees of freedom of the test.          |
| p.value     | The p-value of the test.                     |
| alternative | A character with the alternative hypothesis. |
| method      | A character with the test used.              |
| data.name   | A character vector with two elements.        |

**Author(s)**

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

**References**

[https://en.wikipedia.org/wiki/Matrix\\_normal\\_distribution#Definition](https://en.wikipedia.org/wiki/Matrix_normal_distribution#Definition)

Pocuca N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

**See Also**

[mn.mle](#), [mn.reg](#)

**Examples**

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
Y1 <- MN::rmn(250, M, U, V)
Y2 <- MN::rmn(200, M, U, V)
mn.equaltest(Y1, Y2)
```

---

Log-likelihood ratio test for equality of mean matrices

*Log-likelihood ratio test for equality of mean matrices*

---

**Description**

Log-likelihood ratio test for equality of mean matrices.

**Usage**

```
mn.ttest(Y1, Y2, tol = 1e-6, maxiter = 100)
```

**Arguments**

|         |                                                                                                                                    |
|---------|------------------------------------------------------------------------------------------------------------------------------------|
| Y1      | The observations of the first sample. A list with k1 elements (k1 is the sample size), k1 matrices of dimension $n \times p$ each. |
| Y2      | The observations of the first sample. A list with k2 elements (k2 is the sample size), k2 matrices of dimension $n \times p$ each. |
| tol     | The tolerance value to terminate the iterative algorithm.                                                                          |
| maxiter | The maximum number of iterations allowed.                                                                                          |

**Details**

A log-likelihood ratio test comparing the equality of mean matrices,  $M_1 = M_2$  is performed, and the alternative is that the mean matrices are not equal.

**Value**

This is an "htest" class object. Thus it returns a list including:

|             |                                              |
|-------------|----------------------------------------------|
| statistic   | The test statistic value.                    |
| parameter   | The degrees of freedom of the test.          |
| p.value     | The p-value of the test.                     |
| alternative | A character with the alternative hypothesis. |
| method      | A character with the test used.              |
| data.name   | A character vector with two elements.        |

**Author(s)**

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

**References**

[https://en.wikipedia.org/wiki/Matrix\\_normal\\_distribution#Definition](https://en.wikipedia.org/wiki/Matrix_normal_distribution#Definition)

Pocua N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

**See Also**

[mn.mle](#), [mn.reg](#)

**Examples**

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
Y1 <- MN::rmn(150, M, U, V)
Y2 <- MN::rmn(100, M, U, V)
mn.ttest(Y1, Y2)
```



---

Matrix normal regression

*Matrix normal regression*

---

## Description

Matrix normal regression.

## Usage

```
mn.reg(Y, X, tol = 1e-6, maxiter = 100, Xnew = NULL)
mn.reg2(Y, x, tol = 1e-6, maxiter = 100, xnew = NULL)
mn.reg_null(Y, tol = 1e-6, maxiter = 100)
```

## Arguments

|         |                                                                                                                                                                                           |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Y       | A list with k elements (k is the sample size), k matrices of dimension $n \times p$ each.                                                                                                 |
| X       | A list with the matrix covariates. A list with k elements (k is the sample size), k matrices of dimension $n \times dx$ each, each matrix is the matrix covariate to the matrix response. |
| x       | A matrix with covariates, where the i-th row represents the covariate vector is related to the i-th response matrix Y.                                                                    |
| tol     | The tolerance value to terminate the iterative algorithm.                                                                                                                                 |
| maxiter | The maximum number of iterations allowed.                                                                                                                                                 |
| Xnew    | If you have new X values (list of matrices or a list with a single matrix) and want to predict the values of the response matrices pass them here.                                        |
| xnew    | If you have new x values (matrix or vector) and want to predict the values of the response matrices pass them here.                                                                       |

## Details

The function `mn.reg()` performs matrix normal regression with matrix covariates. The function `mn.reg2()` performs matrix normal regression with a matrix of covariates. The function `mn.reg_null()` performs matrix normal regression with the constant only. This is useful for performing a log-likelihood ratio test of a regression model against the constant term.

## Value

A list including:

|         |                                                                               |
|---------|-------------------------------------------------------------------------------|
| runtime | The runtime required for the whole fitting procedure.                         |
| iters   | The number of iterations required for the estimation of the U and V matrices. |
| loglik  | The log-likelihood value.                                                     |
| B       | The estimated matrix of coefficients.                                         |

|     |                                                                                                              |
|-----|--------------------------------------------------------------------------------------------------------------|
| U   | The estimated covariance matrix associated with the rows, a numerical matrix of dimensions $n \times n$ .    |
| V   | The estimated covariance matrix associated with the columns, a numerical matrix of dimensions $p \times p$ . |
| est | The estimated matrix responses for the new x values, otherwise this is NULL.                                 |

**Author(s)**

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

**References**

[https://en.wikipedia.org/wiki/Matrix\\_normal\\_distribution#Definition](https://en.wikipedia.org/wiki/Matrix_normal_distribution#Definition)

Pocuca N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

**See Also**

[mn.mle](#)

**Examples**

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
Y <- MN::rmn(150, M, U, V)
x <- iris[, 1:2]
colnames(x) <- c("sepal-length", "sepal-width")
mod2 <- mn.reg2(Y, x)
```

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
X <- MN::rmn(150, M/3, U, V)

mod <- mn.reg(Y, X)
```

---

Maximum likelihood estimation of the matrix normal distribution

*Maximum likelihood estimation of the matrix normal distribution*

---

**Description**

Maximum likelihood estimation of the matrix normal distribution.

**Usage**

```
mn.mle(Y, tol = 1e-6)
```

**Arguments**

|     |                                                                                           |
|-----|-------------------------------------------------------------------------------------------|
| Y   | A list with k elements (k is the sample size), k matrices of dimension $n \times p$ each. |
| tol | The tolerance value to terminate the iterative algorithm.                                 |

**Value**

A list including:

|         |                                                                                                              |
|---------|--------------------------------------------------------------------------------------------------------------|
| runtime | The runtime required for the whole fitting procedure.                                                        |
| iters   | The number of iterations required for the estimation of the U and V matrices.                                |
| loglik  | The log-likelihood value.                                                                                    |
| M       | The estimated mean matrix of the distribution, a numerical matrix of dimensions $n \times p$ .               |
| U       | The estimated covariance matrix associated with the rows, a numerical matrix of dimensions $n \times n$ .    |
| V       | The estimated covariance matrix associated with the columns, a numerical matrix of dimensions $p \times p$ . |

**Author(s)**

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

**References**

[https://en.wikipedia.org/wiki/Matrix\\_normal\\_distribution#Definition](https://en.wikipedia.org/wiki/Matrix_normal_distribution#Definition)

Pocuca N., Gallagher M. P., Clark K. M. & McNicholas P. D. (2019). Assessing and Visualizing Matrix Variate Normality. arXiv:1910.02859.

**See Also**

[dmn](#), [rmn](#), [ddplot](#)

**Examples**

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
Y <- rmn(200, M, U, V)
mod <- mn.mle(Y)
```

---

Random matrices simulation from the matrix normal distribution

*Random matrices simulation from the matrix normal distribution*

---

## Description

Random matrices simulation from the matrix normal distribution.

## Usage

```
rmn(k, M, U, V)
```

## Arguments

|   |                                                                                                    |
|---|----------------------------------------------------------------------------------------------------|
| k | The sample size, the number of matrices to simulate.                                               |
| M | The mean matrix of the distribution, a numerical matrix of dimensions $n \times p$ .               |
| U | The covariance matrix associated with the rows, a numerical matrix of dimensions $n \times n$ .    |
| V | The covariance matrix associated with the columns, a numerical matrix of dimensions $p \times p$ . |

## Value

A list with k elements, k matrices of dimension *n times p* each. These are the random matrices drawn from a matrix normal distribution.

## Author(s)

Michail Tsagris.

R implementation and documentation: Michail Tsagris <mtsagris@uoc.gr>.

## References

[https://en.wikipedia.org/wiki/Matrix\\_normal\\_distribution#Definition](https://en.wikipedia.org/wiki/Matrix_normal_distribution#Definition)

## See Also

[dmn](#), [mn.mle](#), [ddplot](#)

## Examples

```
M <- as.matrix(iris[1:8, 1:4])
U <- cov( matrix( rnorm(100 * 8), ncol = 8 ) )
V <- cov( iris[1:50, 1:4] )
X <- rmn(10, M, U, V)
```

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