Robust Support Vector Machines

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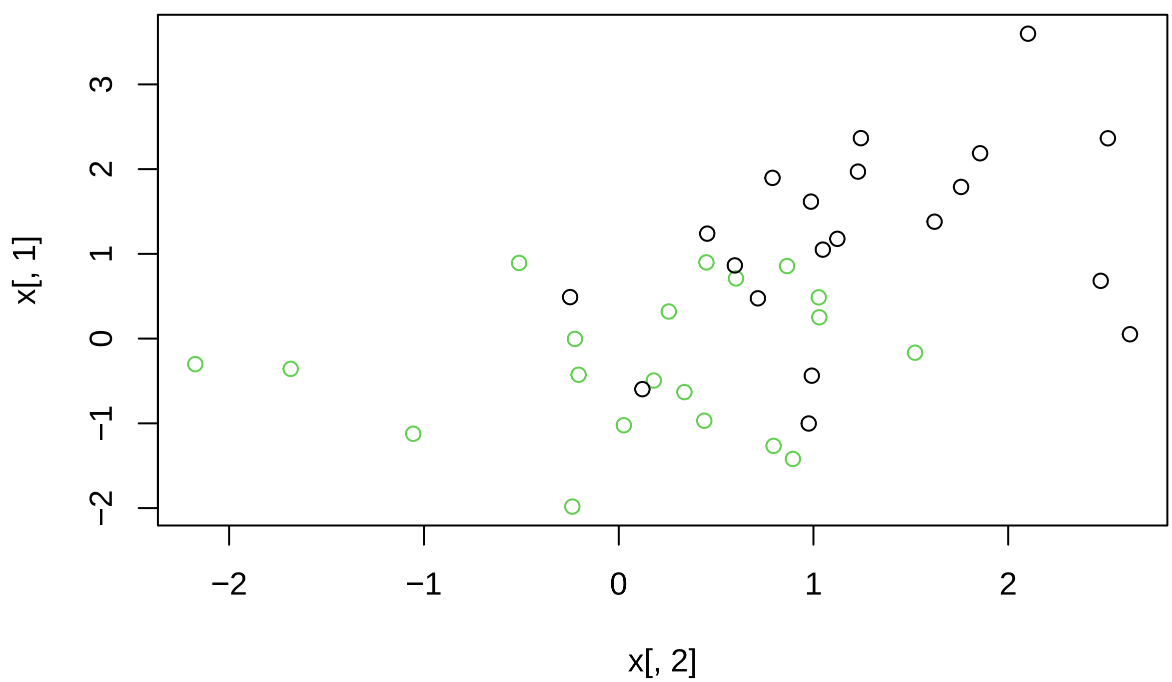
The CC-family contains functions of composite of concave and convex functions. The CC-estimators are derived from minimizing loss functions in the CC-family by the iteratively reweighted convex optimization (IRCO), an extension of the iteratively reweighted least squares (IRLS). The IRCO reduces the weight of the observation that leads to a large loss; it also provides weights to help identify outliers. In the applications of robust support vector machine, the IRCO becomes the iteratively reweighted SVM or IRSVM. See Wang (2020).

Support vector machine classification

```
library("mpath")
library("e1071")
set.seed(1900)
x <- matrix(rnorm(40*2), ncol=2)
y <- c(rep(-1, 20), rep(1, 20))
x[y==1,] <- x[y==1, ] + 1
plot(x[,2],x[,1], col=(2-y))
```



Use the radial kernel SVM for classification.

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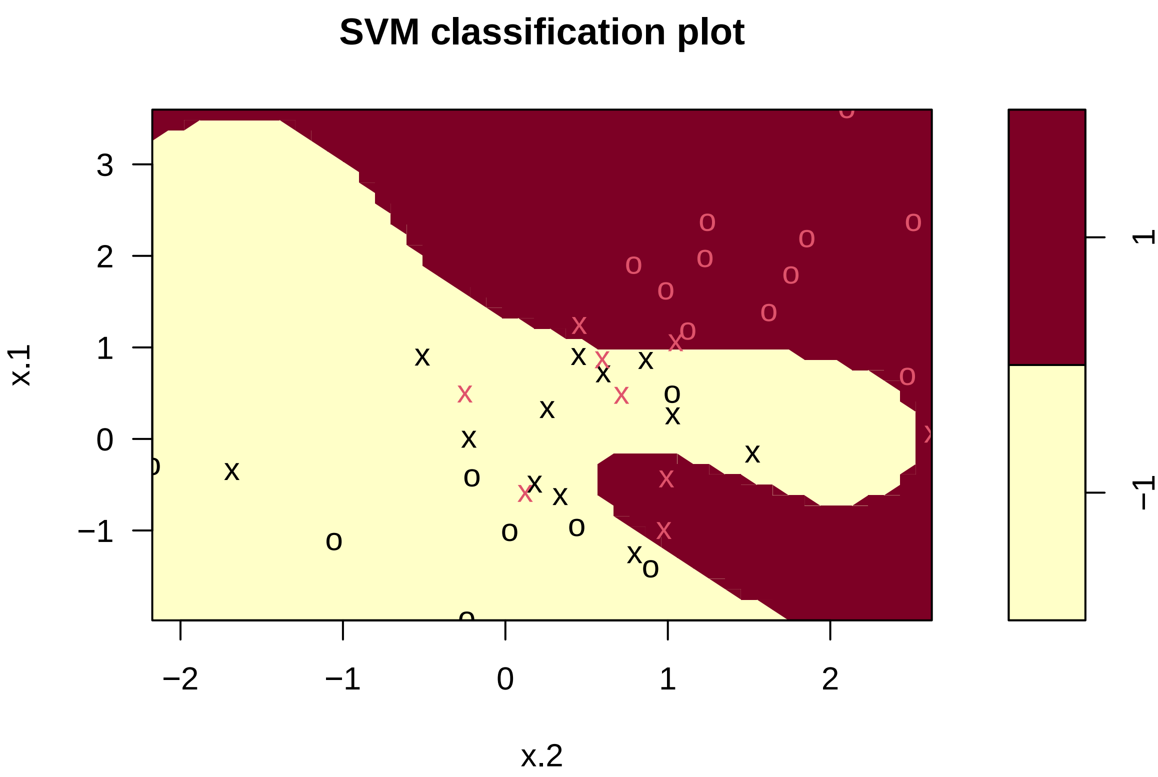
```r
dat <- data.frame(x=x, y=as.factor(y))
svm.model <- svm(y~., data=dat, cost=100, type="C-classification")
summary(svm.model)
```

```
##
## Call:
## svm(formula = y ~ ., data = dat, cost = 100, type = "C-classification")
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  100
##
## Number of Support Vectors:  21
##
##  ( 12 9 )
##
##
## Number of Classes:  2
##
## Levels:
##  -1 1
```

```r
plot(svm.model, dat)
```

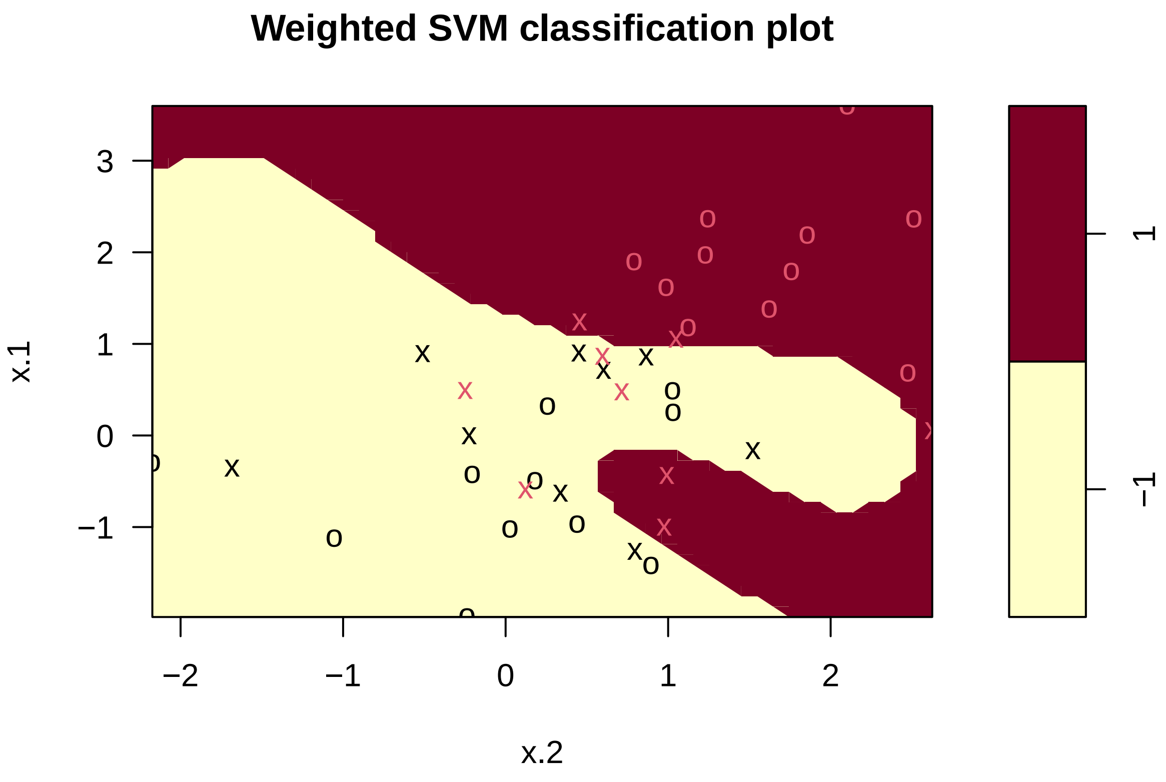


Robust radial kernel SVM for classification.

```r
irsvm.model <- irsvm(y ~ ., data = dat, cost = 100, type="C-classification", cfun="acave",
                     s=1)
summary(irsvm.model)
```

```
##
## Call:
## irsvm.formula(formula = y ~ ., data = dat, cost = 100, type = "C-classification",
##     cfun = "acave", s = 1)
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  100
##
## Number of Support Vectors:  18
##
##  ( 9 9 )
##
##
## Number of Classes:  2
##
## Levels:
##  -1 1
```

```
plot(irsvm.model, dat)
```



Add 15% outliers to the training data, and fit robust SVM, selecting tuning parameters with the cross-validation method.

```
n <- length(y)
nout <- n*0.15
id <- sample(n)[1:nout]
```

```r
cat("id=", id)
```

```
## id= 16 39 30 17 40 25
```

```r
y[id] <- -y[id]
dat2 <- data.frame(x=x, y=as.factor(y))
irsvm.opt <- cv.irsvm(y ~ ., data=dat2, type="C-classification", s=1, cfun="acave",
                      n.cores=2, balance=FALSE)
irsvm.opt$cost
```

```
## [1] 1
```

```r
irsvm.opt$gamma
```

```
## [1] 0.125
```

```r
irsvm.opt$s
```

```
## [1] 1
```

To evaluate prediction, we simulate test data with no outliers.

```r
xtest <- matrix(rnorm(20*2), ncol=2)
ytest <- sample(c(-1,1), 20, rep=TRUE)
xtest[ytest==1, ] <- xtest[ytest==1, ] + 1
testdat <- data.frame(x=xtest, y=as.factor(ytest))
```

Fit a robust SVM model again, with tuning parameters selected by cross-validation, then evaluate prediction accuracy with test data, with 85% accuracy.

```r
irsvm.model1 <- irsvm(y ~ ., data = dat2, cost = irsvm.opt$cost, gamma=irsvm.opt$gamma,
                      s=irsvm.opt$s, cfun="acave", type="C-classification")
summary(irsvm.model1)
```

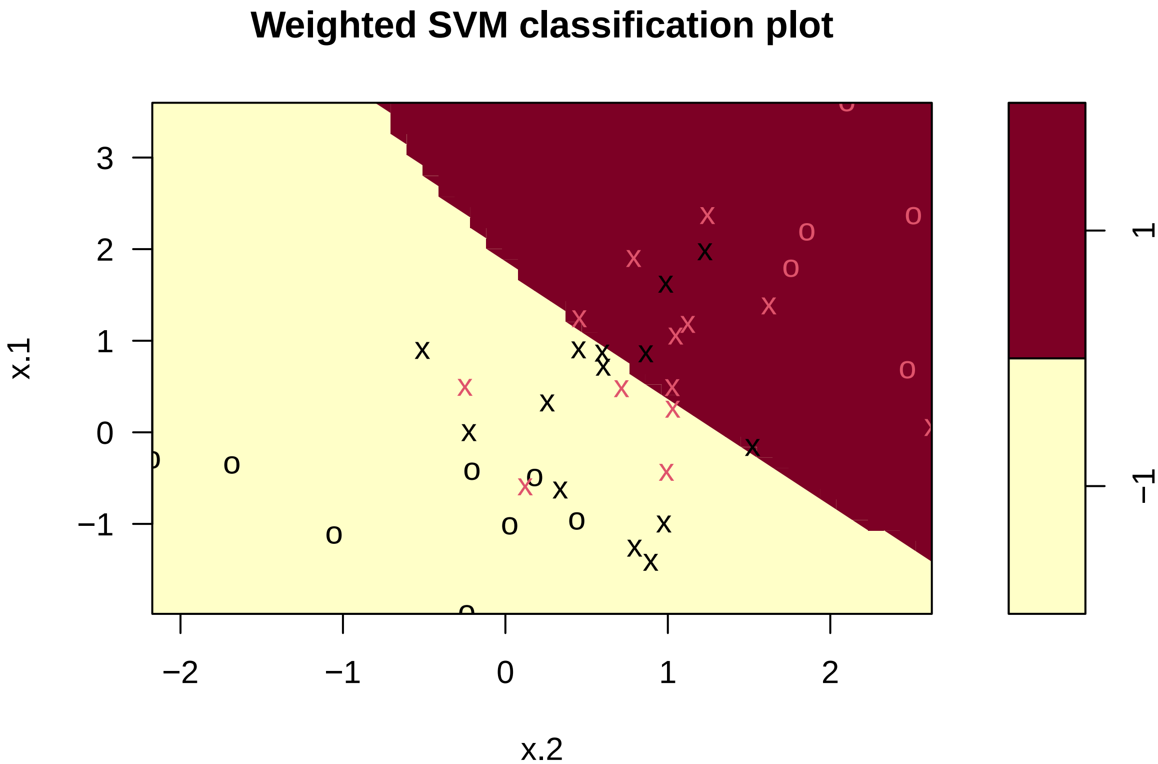
```
##
## Call:
## irsvm.formula(formula = y ~ ., data = dat2, cost = irsvm.opt$cost,
##     gamma = irsvm.opt$gamma, s = irsvm.opt$s, cfun = "acave", type = "C-classification")
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  1
##
## Number of Support Vectors:  27
##
##  ( 14 13 )
##
##
## Number of Classes:  2
##
## Levels:
##  -1 1
```

```r
table(predict=predict(irsvm.model1, xtest), truth=testdat$y)
```

```
##        truth
## predict -1  1
```

```
##       -1  7  2
##       1   1 10
```

```r
plot(irsvm.model1, dat2)
```



Develop a SVM model with training data and evaluate with the test data. The prediction accuracy is 80%.

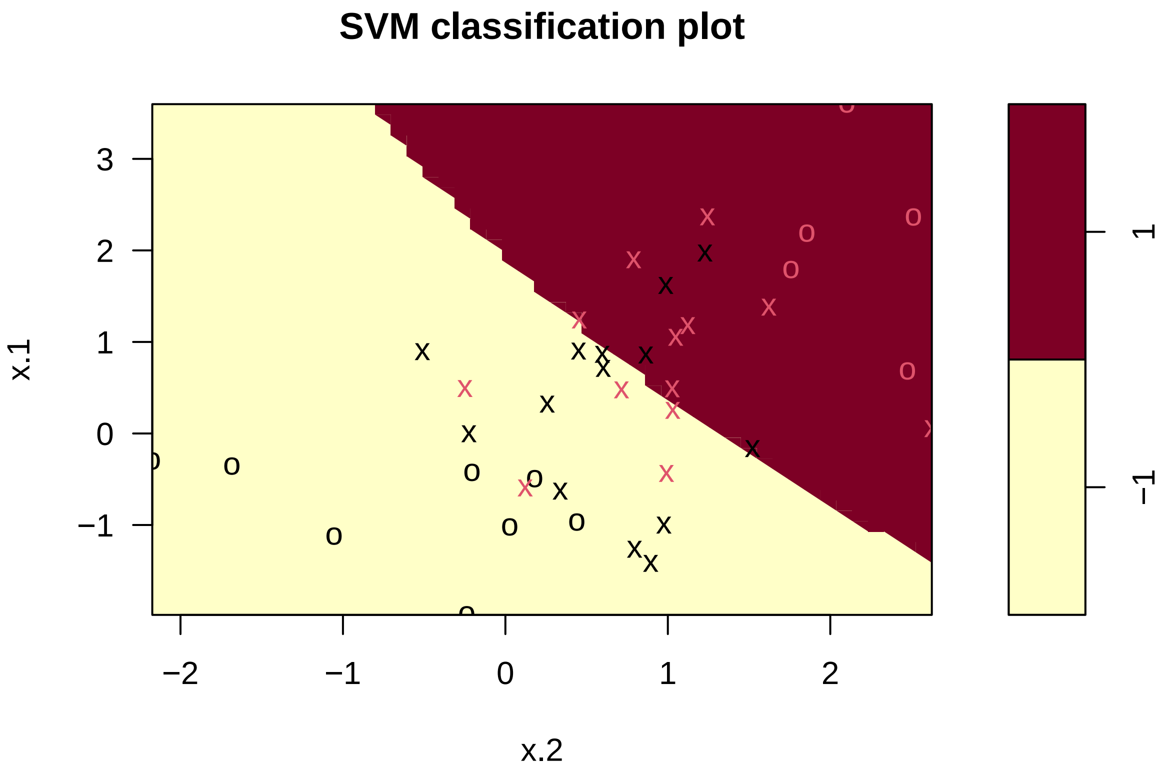
```r
svm.model1 <- svm(y~., data=dat2, cost=irsvm.opt$cost, gamma=irsvm.opt$gamma,
                  type="C-classification")
summary(svm.model1)
```

```
##
## Call:
## svm(formula = y ~ ., data = dat2, cost = irsvm.opt$cost, gamma = irsvm.opt$gamma,
##     type = "C-classification")
##
##
## Parameters:
##    SVM-Type:  C-classification
##  SVM-Kernel:  radial
##        cost:  1
##
## Number of Support Vectors:  27
##
##  ( 14 13 )
##
##
## Number of Classes:  2
##
## Levels:
##  -1 1
```

```r
table(predict=predict(svm.model1, testdat), truth=testdat$y)
```

```
##        truth
## predict -1 1
##      -1  7 3
##      1   1 9
```

```r
plot(svm.model1, dat2)
```



In robust SVM with function `irsvm`, argument `cfun` can be chosen from `"hcave"`, `"acave"`, `"bcave"`, `"ccave"`, `"dcave"`, `"gcave"`, `"tcave"`, `"ecave"`, for a variety of concave functions.

Support vector machine regression

We predict median value of owner-occupied homes in suburbs of Boston. The data can be obtained from the UCI machine learning data repository. There are 506 observations and 13 predictors.

```r
urlname <- "https://archive.ics.uci.edu/ml/"
filename <- "machine-learning-databases/housing/housing.data"
dat <- read.table(paste0(urlname, filename), sep="", header=FALSE)
n <- dim(dat)[1]
p <- dim(dat)[2]
cat("n=",n,"p=", p, "\n")
```

```
## n= 506 p= 14
```

Randomly split the data into 90% of samples for training and 10% of samples as test data.

```r
set.seed(129)
trid <- sample(n)[1:(n*0.9)]
```

```r
traindat <- dat[trid, ]
testdat <- dat[-trid, ]
```

Fit the robust radial kernel irsvm model with truncated ϵ-insensitive loss, i.e., `cfun="tcave"` in function `irsvm`. Root mean squared error on test data is reported. A comprehensive robust irsvm analysis with other types of `cfun` can be found in Wang (2020).

```r
irsvm.model <- irsvm(x=traindat[,-p], y=traindat[,p], cost = 2^3, gamma=2^(-4),
                     epsilon=2^(-4), s=5, cfun="tcave")
summary(irsvm.model)
```

```
##
## Call:
## irsvm.matrix(x = x, y = ..1, cost = ..2, gamma = ..3, epsilon = ..4,
##     s = 5, cfun = "tcave")
##
##
## Parameters:
##    SVM-Type:  eps-regression
##  SVM-Kernel:  radial
##        cost:  8
##       gamma:  0.0625
##     epsilon:  0.0625
##
##
## Number of Support Vectors:  351
```

```r
irsvm.predict <- predict(irsvm.model, testdat[,-p])
mse1 <- mean((testdat[,p] - irsvm.predict)^2)
cat("RMSE with robust SVM", sqrt(mse1))
```

```
## RMSE with robust SVM 2.758136
```

Fit the radial kernel SVM model. The RMSE is larger than the robust SVM, and the model has a larger number of support vectors as well. See the figure below for a comparison.

```r
svm.model <- svm(x=traindat[,-p], y=traindat[,p], cost=2^3, gamma=2^(-4), epsilon=2^(-4))
summary(svm.model)
```

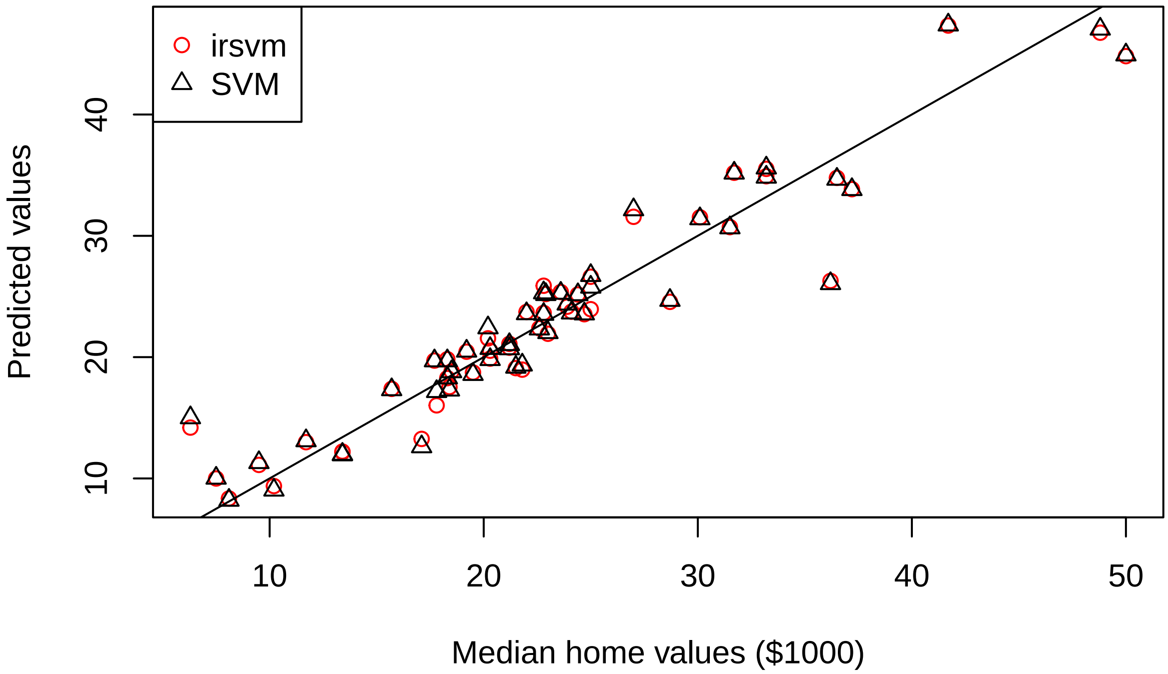
```
##
## Call:
## svm.default(x = traindat[, -p], y = traindat[, p], gamma = 2^(-4),
##     cost = 2^3, epsilon = 2^(-4))
##
##
## Parameters:
##    SVM-Type:  eps-regression
##  SVM-Kernel:  radial
##        cost:  8
##       gamma:  0.0625
##     epsilon:  0.0625
##
##
## Number of Support Vectors:  361
```

```r
svm.predict <- predict(svm.model, testdat[,-p])
mse2 <- mean((testdat[,p] - svm.predict)^2)
```

```r
cat("RMSE with SVM", sqrt(mse2))
```

```
## RMSE with SVM 2.840434
```

```r
plot(testdat[,p], irsvm.predict, col="red", pch=1, ylab="Predicted values",
     xlab="Median home values ($1000)")
points(testdat[,p], svm.predict, col="black", pch=2)
legend("topleft", c("irsvm", "SVM"), col=c("red", "black"), pch=c(1, 2))
abline(coef=c(0, 1))
```



Reference

Wang, Zhu. 2020. "Unified Robust Estimation." *arXiv e-Prints*, October, arXiv:2010.02848. https://arxiv.org/abs/2010.02848.