Package 'hmer'

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```
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Title History Matching and Emulation Package
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URL https://github.com/andy-iskauskas/hmer,
     https://hmer-package.github.io/website/
Description A set of objects and functions for Bayes Linear emulation and history matching.
     Core functionality includes automated training of emulators to data, diagnostic functions
     to ensure suitability, and a variety of proposal methods for generating 'waves' of points.
     For details on the mathematical background, there are many papers available on the topic
     (see references attached to function help files or the below references); for details of
     the functions in this package, consult the manual or help files.
     Iskauskas, A, et al. (2024) <doi:10.18637/jss.v109.i10>.
     Bower, R.G., Goldstein, M., and Vernon, I. (2010) <doi:10.1214/10-BA524>.
     Craig, P.S., Goldstein, M., Seheult, A.H., and Smith, J.A. (1997) <doi:10.1007/978-1-4612-
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Description

Produces summary and plots for diagnostics

Usage

```
analyze_diagnostic(
  in_data,
  output_name,
  targets = NULL,
  plt = interactive(),
  cutoff = 3,
  target_viz = NULL,
  ...
)
```

Arguments

in_data	The data to perform the analysis on
output_name	The name of the output emulated
targets	If required or desired, the targets for the system outputs
plt	Whether or not to plot the analysis
cutoff	The implausibility cutoff for diagnostic 'ce'
target_viz	How to show the targets on the diagnostic plots
	Any other parameters to pass to subfunctions

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Details

Given diagnostic information (almost certainly provided from get_diagnostic), we can plot the results and highlight the points that are worthy of concern or further consideration. Each diagnostic available has a plot associated with it which can be produced here:

Standardized Error: A histogram of standardized errors. Outliers should be considered, as well as whether very many points have either large or small errors.

Comparison Diagnostics: Error bars around points, corresponding to emulator prediction plus or minus emulator uncertainty. A green line indicates where the emulator and simulator prediction would be in complete agreement: error bars that do not overlap with this line (coloured red) are to be considered. Where targets are provided, the colouration is limited only to points where the simulator prediction would be close to the targets.

Classification Error: A point plot comparing emulator implausibility to simulator implausibility, sectioned into regions horizontally and vertically by cutoff. Points that lie in the lower right quadrant (i.e. emulator would reject; simulator would not) should be considered.

This function takes a data.frame that contains the input points, simulator values and, depending on the diagnostic, a set of summary measures. It returns a data.frame of any points that failed the diagnostic.

We may also superimpose the target bounds on the comparison diagnostics, to get a sense of where it is most important that the emulator and simulator agree. The target_viz argument controls this, and has three options: 'interval' (a horizontal interval); 'solid' (a solid grey box whose dimensions match the target region in both vertical and horizontal extent); and 'hatched' (similar to solid, but a semi-transparent box with hatching inside). Any such vizualisation has extent equal to the target plus/minus 4.5 times the target uncertainty. By default, target_viz = NULL, indicating that no superposition is shown.

Value

A data.frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic
```

Other diagnostic functions: classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()

behaviour_plot 5

Description

A simple diagnostic plot that compares the output values to input values, for each possible combination. If emulators are provided, the emulator predictions are plotted; otherwise the model outputs are plotted.

Usage

```
behaviour_plot(
   ems,
   points,
   model = missing(ems),
   out_names = unique(names(collect_emulators(ems))),
   targets = NULL
)
```

Arguments

ems A set of Emulator objects.

points A set of points at which to evaluate the emulator expectation

model If TRUE, use the model outputs; else use emulator expectation

out_names If no emulators are provided, use this argument to indicate outputs.

targets If targets are provided, these are added into the plots.

Details

If emulators are provided, then the points argument is optional: if given then the emulator predictions will correspond to those at the points provided. If no points are provided, 100*d (where d is the number of input parameters) are sampled uniformly from the space and used to predict at.

If no emulators are provided, then points must be provided, along with the names of the outputs to plot; each named output must exist as a column in the points data.frame.

Value

The dependency plots.

See Also

```
Other visualisation tools: diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
behaviour_plot(SIREmulators$ems, model = FALSE)
behaviour_plot(points = SIRSample$training, out_names = names(SIREmulators$ems))
#> Throws a warning
behaviour_plot(SIRMultiWaveEmulators, model = TRUE, targets = SIREmulators$targets)
```

```
bimodal_emulator_from_data
```

Bimodal Emulation

Description

Performs emulation of bimodal outputs and/or systems.

Usage

```
bimodal_emulator_from_data(
   data,
   output_names,
   ranges,
   input_names = names(ranges),
   verbose = interactive(),
   na.rm = FALSE,
   ...
)
```

Arguments

data The data to train emulators on (as in variance_emulator_from_data)

ranges The parameter ranges

input_names The names of the parameters (by default inferred from ranges)

verbose Should status updates be provided?

na.rm Should NA values be removed before training?
... Any other parameters to pass to emulator training

Details

This function is deprecated in favour of using emulator_from_data with argument emulator_type = "multistate". See the associated help file.

In many stochastic systems, particularly disease models, the outputs exhibit bimodality - a familiar example is where a disease either takes off or dies out. In these cases, it is not sensible to emulate the outputs based on all realisations, and instead we should emulate each mode separately.

This function first tries to identify bimodality. If detected, it determines which of the outputs in the data exhibits the bimodality: to these two separate emulators are trained, one to each mode. The

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emulators are provided with any data that is relevant to their training; for example, bimodality can exist in some regions of parameter space but not others. Points where bimodality is present have their realisations allocated between the two modes while points where no bimodality exists have their realisations provided to both modes. Targets that do not exhibit bimodality are trained as a normal stochastic output: that is, using the default of variance_emulator_from_data.

The function also estimates the proportion of realisations in each mode for the set of outputs. This value is also emulated as a deterministic emulator and included in the output.

The output of the function is a list, containing three objects: mode1, mode2, and prop. The first two objects have the form produced by variance_emulator_from_data while prop has the form of an emulator_from_data output.

Value

A list (mode1, mode2, prop) of emulator lists and objects.

Examples

```
# Excessive runtime # Use the stochastic SIR dataset SIR\_ranges \leftarrow list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05)) SIR\_names \leftarrow c("I10", "I25", "I50", "R10", "R25", "R50") b\_ems \leftarrow bimodal\_emulator\_from\_data(SIR\_stochastic\$training, SIR\_names, SIR\_ranges)
```

BirthDeath

Birth-Death Model Results

Description

An RData object containing two data frames. The first consists of ten parameter sets run through a simple, two-parameter, stochastic birth-death model; five of the points have 500 replicates and the other five have only 5 replicates. The second consists of ten further points, each with ten replicates. The objects are denoted training and validation, representing their expected usage.

Usage

BirthDeath

Format

A list of two data.frames training and validation: each data.frame has the following columns:

lambda Birth rate

mu Death rate

Y The number of people at time t = 15

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Details

The initial population for the simulations is 100 people; the model is run until t = 15 to obtain the results to emulate.

classification_diag Classification Diagnostics

Description

Shorthand function for diagnostic test 'ce'.

Usage

```
classification_diag(
  emulator,
  targets,
  validation,
  cutoff = 3,
  plt = interactive()
)
```

Arguments

emulator The emulator in question
targets The output targets
validation The validation set
cutoff The implausibility cutoff
plt Whether to plot or not

Details

For details of the function, see get_diagnostic and for the plot see analyze_diagnostic.

Value

A data.frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic, analyze_diagnostic, validation_diagnostics
Other diagnostic functions: analyze_diagnostic(), comparison_diag(), get_diagnostic(),
individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

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collect_emulators

Collect and order emulators

Description

Manipulates lists (or lists of lists) of emulators into a useable form.

Usage

```
collect_emulators(
  emulators,
  targets = NULL,
  cutoff = 3,
  ordering = c("params", "imp", "volume"),
  sample_size = 200,
  ...
)
```

Arguments

emulators The recursive list of emulators

targets If not NULL, uses implausibility to order the emulators.

cutoff The implausibility cutoff to use (if required)
ordering The order in which to apply the relevant metrics

sample_size The number of points to apply implausibility to (if required)

... Any additional arguments to pass recursively to collect emulators

Details

Most often used as a pre-processing stage for generate_new_design or nth_implausible, this takes a list of emulators in a variety of forms coming from either multiple waves of history matching, hierarchical emulation or bimodal emulation, and arrange them in a form suitable for sequential analysis. Emulators are also ordered by a number of factors: number of parameters, size of the minimum enclosing hyperrectangle, and implausibility (where applicable).

If targets are provided, then the emulators can also be tested on the basis of how restrictive they are: this is included in the ordering. The cutoff by which to make a determination of implausibility for a point is governed by cutoff. The number of points to sample to consider implausibility is chosen by the value of sample_size: higher values are likely to be a more accurate reflection but will take longer.

The weighting of each of the three metrics can be chosen using the ordering argument: metrics with higher weight are closer to the front of the character vector. The metrics are denoted "params" for number of parameters, "imp" for restrictiveness, and "volume" for volume of the hyperrectangle. For instance, a character vector c("volume", "imp") would sort first by volume of the minimum enclosing hyperrectangle, resolve ties by restrictiveness, and not consider the number of parameters.

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Value

A list of emulators with the ordered property described above.

comparison_diag Comparison Diagnostics

Description

Shorthand function for diagnostic test 'cd'.

Usage

```
comparison_diag(emulator, targets, validation, sd = 3, plt = interactive())
```

Arguments

emulator The emulator in question

targets The output targets validation The validation set

sd The range of uncertainty allowed

plt Whether to plot or not

Details

For details of the function, see get_diagnostic and for the plot see analyze_diagnostic.

Value

A data.frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic, analyze_diagnostic, validation_diagnostics
Other diagnostic functions: analyze_diagnostic(), classification_diag(), get_diagnostic(),
individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

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Correlator

Correlation Structure

Description

Creates a correlation structure, with the necessary specifications.

The correlator has three main elements: the type of correlator, the associated hyperparameters, and the nugget term. The nugget term is broadly separate from the other two parameters, being type-independent.

Constructor

Correlator\$new(corr, hp, nug)

Arguments

corr The type of correlation function. This is provided as a string which corresponds exactly with a function - the function should take three arguments x, xp, hp. This gives a correlation function u(x, xp) defined by hyperparameters hp. For a simple example, see exp_sq .

hp The associated hyperparameters needed to define the correlation structure, as a named list. In the case of exp_sq, this is a list of one element, list(theta).

nug The size of the nugget term. In situations where not all variables are active, the main part of u(x) operates only on the active parts, xA. The presence of the nugget term accounts for the fact that points at the same position in the active space need not be at the same position in the full space.

By default, Correlator\$new() initialises with corr = exp_sq, hp = list(theta = 0.1), and nug = 0.

Accessor Methods

get_corr(x, xp = NULL, actives = TRUE) Returns the correlation between two points. If xp is NULL, then this is correlation between a set of points and themselves (i.e. 1 on the diagonal). All variables are assumed to be active unless otherwise stated in actives.

get_hyper_p() Returns the list of hyperparameters.

print() Produces a summary of the correlation structure specification.

Object Methods

set_hyper_p(hp, nugget) Modifies the hyperparameter and/or nugget terms. Returns a new Correlator object.

Options for Correlations

The default choice (and that supported by other functions in this package, particularly emulator_from_data) for the correlation structure is exponential-squared, due to the useful properties it possesses. However, one can manually instantiate a Correlator with a different underlying structure. Built-in alternatives are as follows, as well as whether a form exists for its derivative:

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```
matern the Matérn function (derivative exists)

orn_uhl the Ornstein-Uhlenbeck function (no derivative)

rat_quad the rational quadratic function (derivative exists)
```

One more function, gamma_exp, is available but not directly supported by emulator_from_data, for example, due to its very limited suitability to emulating model outputs. However, this can be used as a test case for writing one's own correlation functions and using them with emulator_from_data.

A user-defined correlation function can be provided to the Correlator: the requirements are that the function accept data.matrix objects as its first and second arguments, and accept a named list of hyperparameters as its third argument, and return a matrix of correlations between rows of the data.matrices. If a derivative also exists, it should take the same name as the correlation function with "_d" appended to it, and the directions to differentiate with respect to should come after the hyperparameter argument. For example, the rational quadratic functions have the form

```
rat_quad(x1, x2, hp = list(alpha, theta))
rat_quad_d(x1, x2, hp = list(alpha, theta), dx1, dx2)
```

If defining a custom correlation function, care should be taken with hyperparameter estimation - see emulator_from_data examples for details.

Examples

```
test_corr <- Correlator$new(nug = 0.1)
test_corr
point1 <- data.frame(a = 0.1, b = 0.2, c = 0.3)
point2 <- data.frame(a = 0.15, b = 0.18, c = 0.295)
test_corr$get_corr(point1) #> 1
test_corr$get_corr(point1, point2) #> 0.6717557
test_corr$get_corr(point1, point2, actives = c(TRUE, TRUE, FALSE)) #> 0.6734372

new_corr <- test_corr$set_hyper_p(list(theta = 0.5), nug = 0.01)
new_corr$get_corr(point1, point2) #> 0.9784845
new_corr$get_corr(point1, point2, actives = c(TRUE, TRUE, FALSE)) #> 0.9785824

mat_corr <- Correlator$new('matern', list(nu = 1.5, theta = 0.5))
mat_corr$get_corr(data.frame(a = c(1, 0.9), b = c(4, 4.2)))</pre>
```

diagnostic_pass

Automated Diagnostics and Modifications

Description

Perform a set of diagnostics on emulators, changing them if needed.

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Usage

```
diagnostic_pass(
  ems,
  targets,
  validation,
  check_output = FALSE,
  verbose = interactive(),
  ...
)
```

Arguments

ems The emulators to consider

targets The output targets to compare implausibility against

validation The set of validation points (either a single data.frame or one per emulator)

check_output Whether to check for suitability of outputs re. targets verbose Whether messages should be printed while running

... Other arguments to pass to helper functions

Details

NOTE: Automated diagnostics are currently only supported for deterministic emulators.

There are a number of different characteristics that emulators might possess that give rise to diagnostic flags. This function collects together some of those whose resulting modifications can be automated. The tests, and consequences, are as follows.

Structured Input Space Looks for errors with dependence on input parameters. If found, the emulator's correlation length is reduced (to a minimum of 1/3);

Structured Output Space Looks for errors with dependence on output value. If found, the training and validation data is resampled and emulators are retrained, to try to incorporate/remove high leverage points;

Misclassification Checks agreement between emulator and simulator implausibility classification. If they do not match, emulator uncertainty is inflated;

Comparison Checks that the emulator predictions agree with the simulator predictions at the validation points, allowing for expected margin of error.

If the automated modifications are not sufficient to remove problems, then offending emulators are removed from the set under consideration. Emulators in this category should be carefully considered and their outputs analyzed: they may require manual determination of the regression surface or additional training points in the neighbourhood of the problematic inputs.

The validation set can also be checked for suitability independent of emulator structure: if check_output = TRUE then the outputs of the validation set will be compared against targets, as well as checking the implausibility of the points with respect to the emulators. If any outputs are consistent underor over-estimates, or if all points are to be ruled out as implausible, the emulators corresponding to these outputs are removed. This option should be used with care: such a situation could be informative for considering the model structure and whether one should expect a match to observational data.

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Value

A collection of modified emulators, potentially a subset of the original collection

Examples

```
new_ems <- diagnostic_pass(SIREmulators$ems, SIREmulators$targets, SIRSample$validation)</pre>
```

diagnostic_wrap

Diagnostic plots for wave outputs

Description

A wrapper function for the set of diagnostic plots for multiple waves.

Usage

```
diagnostic_wrap(
  waves,
  targets,
  output_names = names(targets),
  input_names = names(waves[[1]])[!names(waves[[1]]) %in% names(targets)],
  directory = NULL,
  s.heights = rep(1000, 4),
  s.widths = s.heights,
  include.norm = TRUE,
  include.log = TRUE,
  ...
)
```

Arguments

```
waves
                  The wave points, as a list of data.frames.
targets
                  The output targets.
output_names
                  The outputs to plot.
input_names
                  The inputs to plot.
directory
                  The location of files to be saved (if required).
s.heights
                  The heights of the saved pngs (if directory is not NULL).
s.widths
                  The widths of the saved pngs (if directory is not NULL).
                  Should normalized versions of simulator_plot and wave_dependencies be made?
include.norm
include.log
                  Should the log-scale version of simulator_plot be made?
                  Optional parameters (eg p_size, l_wid, ...)
```

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Details

The functions simulator_plot, wave_points, wave_points, and wave_dependencies are called, one after the other, to allow diagnosis of waves of emulation.

The directory option should be used as follows. If the desired location is in fact a folder, it should end in "/"; if instead the structure requires each plot to be saved with a prefix, then it should be provided. For example, directory = "Plots/" in the first event or directory = "Plots/unique-identifier" in the second event.

Value

The set of plots (either into console or saved).

See Also

```
Other visualisation tools: behaviour_plot(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
diagnostic_wrap(SIRMultiWaveData, SIREmulators$targets)
diagnostic_wrap(SIRMultiWaveData, SIREmulators$targets,
input_names = c('aSI', 'aIR'), output_names = c('nI', 'nR'),
p_size = 0.8, l_wid = 0.8, wave_numbers = 1:3, zero_in = FALSE, surround = TRUE)
```

directional_deriv

Derivative inner product

Description

Find the (uncertainty modified) inner product between the derivative at a point x and a proposed direction v.

Usage

```
directional\_deriv(em, x, v, sd = NULL, ...)
```

Arguments

em	The emulator in question
Х	The point in input space to evaluate at
V	The direction to assess
sd	How many standard deviations to consider.
	Additional arguments to pass through (eg local.var to the emulator functions)

Details

Given a point x and a direction v, we find the overlap between E[f'(x)] and v. The emulated derivative has uncertainty associated with it: the variance is taken into account using $v^T Var[f'(x)]v$.

If sd == NULL, then only the (normed) overlap between the derivative and the direction vector is returned. Otherwise a pair of values are returned: these are the normed overlap plus or minus sd times the uncertainty.

This function is concerned with ascertaining whether a direction is oriented in the direction of the emulator gradient, subject to the uncertainty around the estimate of the derivative. It allows for a consideration of "emulated gradient descent".

Value

Either a single numeric or a pair of numerics (see description)

Examples

```
directional_deriv(SIREmulators$ems[[1]], SIRSample$validation[1,], c(1,1,1))
```

Description

Proposes a new point by applying 'emulated gradient descent' on an existing point.

Usage

```
directional_proposal(
  ems,
  x,
  targets,
  accept = 2,
  hstart = 1e-04,
  hcutoff = 1e-09,
  iteration.measure = "exp",
  iteration.steps = 100,
  nv = 500
)
```

Arguments

ems The emulators to evaluate with respect to.

x The original point.

targets The list of emulator targets.

accept The implausibility below which we allow an output to worsen.

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hstart The initial step size.

hcutoff The minimum allowed step size.

iteration.measure

Either 'exp' for expectation or 'imp' for implausibility.

iteration.steps

The number of allowed iterations.

nv The number of directions on the n-sphere to try.

Details

Given a point (preferably close to the implausibility boundary) x, we can calculate the emulated gradient at this point for each emulator. If the estimate of the expectation at this point for a given emulator is larger than the target value, then we would like to move in the direction of greatest decrease for this emulator, and conversely for an estimate of the expectation that's smaller than the target value. The combination of this information for every emulator under consideration defines a preferred set of directions of travel from this point.

We may try to find a shared direction which improves (or at least does not worsen) all emulator evaluations. If a point is already well inside the implausibility boundary for a given output (where 'well inside' is defined by the value of accept), we may allow this output to worsen in order to improve the others.

Provided a shared direction, v, can be identified, we iteratively move in this direction. Define the new proposed point x' = x + h*v, where h is a step-size given by hstart. Compare the summary statistic (either expectational difference or implausibility) to that provided by the original point; if the new point gives improvement, then continue to move in this direction until no further improvement is possible for this step-size. The step-size is reduced (up to a minimum of hcutoff) and the process is repeated. Only finitely many iteration steps are permitted; this can be tuned by supplying a value of iteration.steps.

Value

Either a new proposal point, or the original point if an improvement could not be found.

Examples

```
# Take a point from the SIR system at later waves with low (but >3) implausibility
start_point <- SIRMultiWaveData[[2]][90,1:3]
ems <- SIRMultiWaveEmulators[[3]]
targs <- SIREmulators$targets
# Using expected error as measure
new_point1 <- directional_proposal(ems, start_point, targs, iteration.steps = 50,
nv = 100)
# Using implausibility as measure
new_point2 <- directional_proposal(ems, start_point, targs, iteration.measure = 'imp',
iteration.steps = 50, nv = 100)
all_points <- do.call('rbind.data.frame', list(start_point, new_point1, new_point2))
nth_implausible(ems, all_points, targs)</pre>
```

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effect_strength

Find Effect Strength of Active Variables

Description

Collates the linear and quadratic contributions of the active variables to the global emulators' behaviour

Usage

```
effect_strength(
  ems,
  plt = interactive(),
  line.plot = FALSE,
  grid.plot = FALSE,
  labels = TRUE,
  quadratic = TRUE,
  xvar = TRUE
)
```

Arguments

ems The Emulator object(s) to be analysed.

plt Should the results be plotted? line.plot Should a line plot be produced?

grid.plot Should the effect strengths be plotted as a grid? labels Whether or not the legend should be included.

quadratic Whether or not quadratic effect strength should be calculated.

xvar Should the inputs be used on the x-axis?

Details

For a set of emulators, it can be useful to see the relative contributions of various parameters to the global part of the emulator (i.e. the regression surface). This function extracts the relevant information from a list of emulator objects.

The parameter quadratic controls whether quadratic effect strength is calculated and plotted (an unnecessary plot if, say, linear emulators have been trained). The remaining options control visual aspects of the plots: line.plot determines whether a line or bar (default) plot should be produced, grid.plot determines whether the results are plotted as a graph or a grid, and labels determines if a legend should be provided with the plot (for large numbers of emulators, it is advisable to set this to FALSE).

Value

A list of data.frames: the first is the linear strength, and the second quadratic.

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See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
effect <- effect_strength(SIREmulators$ems)
effect_line <- effect_strength(SIREmulators$ems, line.plot = TRUE)
effect_grid <- effect_strength(SIREmulators$ems, grid.plot = TRUE)</pre>
```

Emulator

Bayes Linear Emulator

Description

Creates a univariate emulator object.

The structure of the emulator is f(x) = g(x) * beta + u(x), for regression functions g(x), regression coefficients beta, and correlation structure u(x). An emulator can be created with or without data; the preferred method is to create an emulator based on prior specifications in the absence of data, then use that emulator with data to generate a new one (see examples).

Constructor

```
Emulator$new(basis_f, beta, u, ranges, ...)
```

Arguments

Required:

basis_f A list of basis functions to be used. The constant function function(x) 1 should be provided as the first element.

beta The specification for the regression parameters. This should be provided in the form list(mu, sigma), where mu are the expectations of the coefficients (aligning with the ordering of basis_f) and sigma the corresponding covariance matrix.

u The specifications for the correlation structure. This should be specified in the form list(sigma, corr), where sigma is a single-valued object, and corr is a Correlator object.

ranges A named list of ranges for the input parameters, provided as a named list of length-two numeric vectors.

Optional:

data A data.frame consisting of the data with which to adjust the emulator, consisting of input values for each parameter and the output.

out_name The name of the output variable.

a_vars A logical vector indicating which variables are active for this emulator.

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discs Model discrepancies: does not include observational error. Ideally split into list(internal = ..., external = ...).

Internal:

model If a linear model, or otherwise, has been fitted to the data, it lives here.

original_em If the emulator has been adjusted, the unadjusted Emulator object is stored, for use of set_sigma or similar.

multiplier A multiplicative factor to be applied to u_sigma. Typically equal to 1, unless changes have been made by, for example, mult_sigma.

Constructor Details

The constructor must take, as a minimum: a list of vectorised basis functions, whose length is equal to the number of regression coefficients; a correlation structure, which can be non-stationary; and the parameter ranges, used to scale all inputs to the range [-1,1].

The construction of a correlation structure is detailed in the documentation for Correlator.

Accessor Methods

get_exp(x, include_c) Returns the emulator expectation at a point, or at a collection of points. If include_c = FALSE, the contribution made by the correlation structure is not included.

get_cov(x, xp = NULL, full = FALSE, include_c) Returns the covariance between collections of points x and xp. If xp is not supplied, then this is equivalent to $get_cov(x, x, ...)$; if full = TRUE, then the full covariance matrix is calculated - this is FALSE by default due to most built-in uses requiring only the diagonal terms, and allows us to take advantage of computational tricks for efficiency.

implausibility(x, z, cutoff = NULL) Returns the implausibility for a collection of points x. The implausibility is the distance between the emulator expectation and a desired output value, weighted by the emulator variance and any external uncertainty. The target, z, should be specified as a named pair list(val, sigma), or a single numeric value. If cutoff = NULL, the output is a numeric I; if cutoff is a numeric value, then the output is boolean corresponding to I <= cutoff.

 $get_exp_d(x, p)$ Returns the expectation of the derivative of the emulated function, E[f'(x)]. Similar in structure to get_exp but for the additional parameter p, which indicates which of the input dimensions the derivative is performed with respect to.

 $get_cov_d(x, p1, xp = NULL, p2 = NULL, full = FALSE)$ Returns the variance of the derivative of the emulated function, Var[f'(x)]. The arguments are similar to that of get_cov , but for the addition of parameters p1 and p2, which indicate the derivative directions. Formally, the output of this function is equivalent to Cov[df/dp1, df/dp2].

print(...) Returns a summary of the emulator specifications.

plot(...) A wrapper for emulator_plot for a single Emulator object.

Object Methods

adjust(data, out_name) Performs Bayes Linear Adjustment, given data. The data should contain all input parameters, even inactive ones, and the single output that we wish to emulate. adjust creates a new Emulator object with the adjusted expectation and variance resulting from Bayes Linear adjustment, allowing for the requisite predictions to be made using get_exp and get_cov.

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set_sigma(sigma) Modifies the (usually constant) global variance of the correlation structure, Var[u(X)]. If the emulator has been trained, the original emulator is modified and Bayes Linear adjustment is again performed.

mult_sigma(m) Modifies the global variance of the correlation structure via a multiplicative factor. As with set_sigma, this change will chain through any prior emulators if the emulator in question is Bayes Linear adjusted.

set_hyperparams(hp, nugget) Modifies the underlying correlator for u(x). Behaves in a similar way to set_sigma as regards trained emulators. See the Correlator documentation for details of hp and nugget.

References

Goldstein & Wooff (2007) <ISBN: 9780470065662> Craig, Goldstein, Seheult & Smith (1998) <doi:10.1111/1467-9884.00115>

Examples

```
basis_functions <- list(function(x) 1, function(x) x[[1]], function(x) x[[2]])
beta <- list(mu = c(1,2,3),
             sigma = matrix(c(0.5, -0.1, 0.2, -0.1, 1, 0, 0.2, 0, 1.5), nrow = 3))
u <- list(mu = function(x) 0, sigma = 3, corr = Correlator$new('exp_sq', list(theta = 0.1)))
ranges <- list(a = c(-0.5, 0.5), b = c(-1, 2))
em <- Emulator$new(basis_functions, beta, u, ranges)</pre>
# Individual evaluations of points
# Points should still be declared in a data.frame
em\$get_exp(data.frame(a = 0.1, b = 0.1)) #> 0.6
em\$get\_cov(data.frame(a = 0.1, b = 0.1)) #> 9.5
# 4x4 grid of points
sample_points <- expand.grid(a = seq(-0.5, 0.5, length.out = 4), b = seq(-1, 2, length.out = 4))
em$get_exp(sample_points) # Returns 16 expectations
em$get_cov(sample_points) # Returns 16 variances
sample_points_2 <- expand.grid(a = seq(-0.5, 0.5, length.out = 3),
                                b = seq(-1, 2, length.out = 4))
em$get_cov(sample_points, xp = sample_points_2, full = TRUE) # Returns a 16x12 matrix of covariances
fake_data <- data.frame(a = runif(10, -0.5, 0.5), b = runif(10, -1, 2))
fake_data$c <- fake_data$a + 2*fake_data$b
newem <- em$adjust(fake_data, 'c')</pre>
all(round(newem$get_exp(fake_data[,names(ranges)]),5) == round(fake_data$c,5)) #>TRUE
matern_em < -Emulator new(basis_f = c(function(x) 1, function(x) x[[1]], function(x) x[[2]]),
beta = list(mu = c(1, 0.5, 2), sigma = diag(0, nrow = 3)),
 u = list(corr = Correlator$new('matern', list(nu = 1.5, theta = 0.4))),
 ranges = list(x = c(-1, 1), y = c(0, 3)))
matern_em\$get_exp(data.frame(x = 0.4, y = 2.3))
newem_data <- Emulator$new(basis_functions, beta, u, ranges, data = fake_data)</pre>
all(round(newem$get_exp(fake_data[,names(ranges)]),5)
   == round(newem_data$get_exp(fake_data[,names(ranges)]), 5)) #>TRUE
```

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```
newem$get_exp_d(sample_points, 'a')
newem$get_cov_d(sample_points, 'b', p2 = 'a')
```

emulator_from_data

Generate Emulators from Data

Description

Given data from simulator runs, generates a set of Emulator objects, one for each output.

Usage

```
emulator_from_data(
  input_data,
  output_names,
  ranges,
  input_names = names(ranges),
  emulator_type = NULL,
  specified_priors = NULL,
  order = 2,
  beta.var = FALSE,
  corr_name = "exp_sq",
  adjusted = TRUE,
  discrepancies = NULL,
  verbose = interactive(),
  na.rm = FALSE,
  check.ranges = TRUE,
  targets = NULL,
  has.hierarchy = FALSE,
  covariance_opts = NULL,
)
```

Arguments

input_data	Required. A	A data.frame containing parameter ar	nd output values
------------	-------------	--------------------------------------	------------------

output_names Required. A character vector of output names

ranges Required if input_names is not given. A named list of input parameter ranges

input_names Required if ranges is not given. The names of the parameters

emulator_type Selects between deterministic, variance, covariance, and multistate emulation

specified_priors

A collection of user-determined priors (see description)

order To what polynomial order should regression surfaces be fitted?
beta.var Should uncertainty in the regression coefficients be included?
corr_name If not exp_sq, the name of the correlation structures to fit

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adjusted Should the return emulators be Bayes linear adjusted?

discrepancies Any known internal or external discrepancies of the model

verbose Should status updates be provided?

na.rm If TRUE, removes output values that are NA

check.ranges If TRUE, modifies ranges to a conservative minimum enclosing hyperrectangle

targets If provided, outputs are checked for consistent over/underestimation has.hierarchy Internal - distinguishes deterministic from hierarchical emulators

covariance_opts

User-specified options for emulating covariance matrices

... Any additional parameters for custom correlators or additional verbosity options

Details

Many of the parameters that can be passed to this function are optional: the minimal operating example requires input_data, output_names, and one of ranges or input_names. If ranges is supplied, the input names are intuited from that list, data.frame, or data.matrix; if only input_names is supplied, then ranges are assumed to be [-1, 1] for each input.

The ranges can be provided in a few different ways: either as a named list of length-2 numeric vectors (corresponding to upper and lower bounds for each parameter); as a data.frame with 2 columns and each row corresponding to a parameter; or as a data.matrix defined similarly as the data.frame. In the cases where the ranges are provided as a data.frame or data.matrix, the row.names of the data object must be provided, and a warning will be given if not.

If the set (input_data, output_names, ranges) is provided and nothing else, then emulators are fitted as follows. The basis functions and associated regression coefficients are generated using linear regression up to quadratic order, allowing for cross-terms. These regression parameters are assumed 'known'.

The correlation function c(x, x') is assumed to be exp_sq and a corresponding Correlator object is created. The hyperparameters of the correlation structure are determined using a constrained maximum likelihood argument. This determines the variance, correlation length, and nugget term.

The maximum allowed order of the regression coefficients is controlled by order; the regression coefficients themselves can be deemed uncertain by setting beta.var = TRUE (in which case their values can change in the hyperparameter estimation); the hyperparameter search can be overridden by specifying ranges for each using hp_range.

In the presence of expert beliefs about the structure of the emulators, information can be supplied directly using the specified_priors argument. This can contain specific regression coefficient values beta and regression functions func, correlation structures u, hyperparameter values hyper_p and nugget term values delta.

Some rudimentary data handling functionality exists, but is not a substitute for sense-checking input data directly. The na.rm option will remove rows of training data that contain NA values if true; the check.ranges option allows a redefinition of the ranges of input parameters for emulator training if true. The latter is a common practice in later waves of emulation in order to maximise the predictive power of the emulators, but should only be used if it is believed that the training set provided is truly representative of and spans the full space of interest.

Various different classes of emulator can be created using this function, depending on the nature of the model. The emulator_type argument accepts a few different options:

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"variance" Create emulators for the mean and variance surfaces, for each stochastic output

The "default" behaviour will apply if the emulator_type argument is not supplied, or does not match any of the above options. If the data provided looks to display stochasticity, but default behaviour is used, a warning will be generated and only the first model result for each individual parameter set will be used in training.

For examples of this function's usage (including optional argument behaviour), see the examples.

Value

An appropriately structured list of Emulator objects

Examples

```
# Deterministic: use the SIRSample training dataset as an example.
ranges <- list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05))
out_vars <- c('nS', 'nI', 'nR')
ems_linear <- emulator_from_data(SIRSample$training, out_vars, ranges, order = 1)</pre>
ems_linear # Printout of the key information.
# Stochastic: use the BirthDeath training dataset
v_ems <- emulator_from_data(BirthDeath$training, c("Y"),</pre>
list(lambda = c(0, 0.08), mu = c(0.04, 0.13)), emulator_type = 'variance')
# If different specifications are wanted for variance/expectation ems, then
# enter a list with entries 'variance', 'expectation'. Eg corr_names
v_ems_corr <- emulator_from_data(BirthDeath$training, c("Y"),</pre>
list(lambda = c(0, 0.08), mu = c(0.4, 0.13)), emulator_type = 'variance',
corr_name = list(variance = "matern", expectation = "exp_sq")
 # Excessive runtime
 ems_quad <- emulator_from_data(SIRSample$training, out_vars, ranges)</pre>
 ems_quad # Now includes quadratic terms
 ems_cub <- emulator_from_data(SIRSample$training, out_vars, ranges, order = 3)</pre>
 ems_cub # Up to cubic order in the parameters
 ems_unadjusted <- emulator_from_data(SIRSample$training, out_vars, ranges, adjusted = FALSE)</pre>
 ems_unadjusted # Looks the same as ems_quad, but the emulators are not Bayes Linear adjusted
 # Reproduce the linear case, but with slightly adjusted beta values
 basis_f <- list(</pre>
  c(function(x) 1, function(x) x[[1]], function(x) x[[2]]),
  c(function(x) 1, function(x) x[[1]], function(x) x[[2]]),
  c(function(x) 1, function(x) x[[1]], function(x) x[[3]])
 beta_val <- list(
```

[&]quot;covariance Create emulators for the mean surface, and a covariance matrix for the variance surface

[&]quot;multistate" Create sets of emulators per output for multistate stochastic systems

[&]quot;default" Deterministic emulators with no covariance structure

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```
list(mu = c(550, -400, 250)),
 list(mu = c(200, 200, -300)),
 list(mu = c(200, 200, -50))
ems_custom_beta <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
 specified_priors = list(func = basis_f, beta = beta_val)
# Custom correlation functions
corr_structs <- list(</pre>
 list(sigma = 83, corr = Correlator$new('exp_sq', list(theta = 0.5), nug = 0.1)),
 list(sigma = 95, corr = Correlator$new('exp_sq', list(theta = 0.4), nug = 0.25)),
list(sigma = 164, corr = Correlator$new('matern', list(theta = 0.2, nu = 1.5), nug = 0.45))
ems_custom_u <- emulator_from_data(SIRSample$training, out_vars, ranges,</pre>
specified_priors = list(u = corr_structs))
# Allowing the function to choose hyperparameters for 'non-standard' correlation functions
ems_matern <- emulator_from_data(SIRSample$training, out_vars, ranges, corr_name = 'matern')
# Providing hyperparameters directly
matern_hp <- list(</pre>
 list(theta = 0.8, nu = 1.5),
 list(theta = 0.6, nu = 2.5),
 list(theta = 1.2, nu = 0.5)
)
ems_matern2 <- emulator_from_data(SIRSample$training, out_vars, ranges, corr_name = 'matern',
 specified_priors = list(hyper_p = matern_hp))
# "Custom" correaltion function with user-specified ranges: gamma exponential
# Any named, defined, correlation function can be passed. See Correlator documentation
ems_gamma <- emulator_from_data(SIRSample$training, out_vars, ranges, corr_name = 'gamma_exp',
 specified_priors = list(hyper_p = list(gamma = c(0.01, 2), theta = c(1/3, 2))))
# Multistate emulation: use the stochastic SIR dataset
SIR_names <- c("I10", "I25", "I50", "R10", "R25", "R50")
b_ems <- emulator_from_data(SIR_stochastic$training, SIR_names,</pre>
 ranges, emulator_type = 'multistate')
# Covariance emulation, with specified non-zero matrix elements
which_cov <- matrix(rep(TRUE, 16), nrow = 4)</pre>
which_cov[2,3] \leftarrow which_cov[3,2] \leftarrow which_cov[1,4] \leftarrow which_cov[4,1] \leftarrow FALSE
c_ems <- emulator_from_data(SIR_stochastic$training, SIR_names[-c(3,6)], ranges,</pre>
 emulator_type = 'covariance', covariance_opts = list(matrix = which_cov))
```

emulator_plot

Plot Emulator Outputs

Description

A function for plotting emulator expectations, variances, and implausibilities

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Usage

```
emulator_plot(
  ems,
  plot_type = "exp",
  ppd = 30,
  targets = NULL,
  cb = FALSE,
  params = NULL,
  fixed_vals = NULL,
  nth = 1,
  imp_breaks = NULL,
  include_legend = TRUE
)
```

Arguments

ems An Emulator object, or a list thereof.

plot_type The statistic to plot (see description or examples).

ppd The number of points per plotting dimension

targets If required, the targets from which to calculate implausibility

cb A boolean representing whether a colourblind-friendly plot is produced.

params Which two input parameters should be plotted?

fixed_vals For fixed input parameters, the values they are held at.

nth If plotting nth maximum implausibility, which level maximum to plot.

imp_breaks If plotting nth maximum implausibility, defines the levels at which to draw con-

tours.

include_legend For multiple plots, should a combined legend be appended?

Details

Given a single emulator, or a set of emulators, the emulator statistics can be plotted across a twodimensional slice of the parameter space. Which statistic is plotted is determined by plot_type: options are 'exp', 'var', 'sd', 'imp', and 'nimp', which correspond to expectation, variance, standard deviation, implausibility, and nth-max implausibility.

By default, the slice varies in the first two parameters of the emulators, and all other parameters are taken to be fixed at their mid-range values. This behaviour can be changed with the params and fixed_vals parameters (see examples).

If the statistic is 'exp', 'var' or 'sd', then the minimal set of parameters to pass to this function are ems (which can be a list of emulators or a single one) and plot_type. If the statistic is 'imp' or 'nimp', then the targets must be supplied - it is not necessary to specify the individual target for a single emulator plot. If the statistic is 'nimp', then the level of maximum implausibility can be chosen with the parameter nth.

Implausibility plots are typically coloured from green (low implausibility) to red (high implausibility): a colourblind-friendly option is available and can be turned on by setting cb = TRUE.

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The granularity of the plot is controlled by the ppd parameter, determining the number of points per dimension in the grid. For higher detail, at the expense of longer computing time, increase this value. The default is 30.

Value

A ggplot object, or collection thereof.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
# Reducing ppd to 10 for speed.
emulator_plot(SIREmulators$ems, ppd = 10)
emulator_plot(SIREmulators$ems$nS, ppd = 10)
emulator_plot(SIREmulators$ems, plot_type = 'var', ppd = 10, params = c('aIR', 'aSR'))
# Excessive runtime
  emulator_plot(SIREmulators$ems, plot_type = 'imp', ppd = 10,
    targets = SIREmulators$targets,
    fixed_vals = list(aSR = 0.02))
  emulator_plot(SIREmulators$ems, plot_type = 'nimp', cb = TRUE,
    targets = SIREmulators$targets, nth = 2, ppd = 10)
```

exp_sq

Exponential squared correlation function

Description

For points x, xp and a correlation length theta, gives the exponent of the squared distance between x and xp, weighted by theta squared.

Usage

```
exp_sq(x, xp, hp)
```

Arguments

x	A data.frame of rows corresponding to position vectors
хр	A data.frame of rows corresponding to position vectors
hp	The hyperparameter theta (correlation length)

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Value

The exponential-squared correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

```
exp_sq(data.frame(a=1), data.frame(a=2), list(theta = 0.1))
#> 3.720076e-44
exp_sq(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(theta = 0.2))
#> 3.266131e-13
```

full_wave

Automatic Wave Calculation

Description

Performs a full wave of emulation and history matching, given data.

Usage

```
full_wave(
   data,
   ranges,
   targets,
   old_emulators = NULL,
   prop_train = 0.7,
   cutoff = 3,
   nth = 1,
   verbose = interactive(),
   n_points = nrow(data),
   ...
)
```

Arguments

data The data to train with.

ranges The ranges of the input parameters targets The output targets to match to.

 $\hbox{old_emulators} \quad \text{Any emulators from previous waves}.$

prop_train What proportion of the data is used for training.

cutoff The implausibility cutoff for point generation and diagnostics.

nth The level of maximum implausibility to consider.

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verbose	Should progress be printed to console?
n_points	The number of points to generate from generate_new_design.
	Any arguments to be passed to emulator from data.

Details

This function uses all of the functionality from the package in a relatively conservative form. The function performs the following steps:

- 1) Split the data into a training set and a validation set, where prop_train indicates what proportion of the data is used to train.
- 2) Perform emulator training using emulator_from_data. If a more involved specification is desired, optional arguments can be passed to emulator_from_data using the . . . argument.
- 3) Perform diagnostics on the trained emulators, removing emulators that do not display acceptable performance. Global emulator variance may also be modified to ensure that none of the emulators demonstrate misclassification errors (from classification_diag).
- 4) Ordering the remaining emulators from most restrictive to least restrictive on the dataset provided at this wave. Some point generation mechanisms terminate early if a point is ruled out by a single emulator, so the ordering ensures this happens earlier rather than later.
- 5) Generate the new points using the default method of generate_new_design, using the normal procedure (for details, see the description for generate_new_design). By default, it generates the same number of points as it was provided to train and validate on.

If the parameter old_emulators is provided, this should be a list of emulators used at all previous waves - for example if full_wave is used to do a second wave of history matching, then old_emulators would contain the list of first-wave emulators.

The function returns a list of two objects: emulators corresponding to this wave's emulators, and points corresponding to the new proposed points. The points can then be put into the simulator to generate runs for a subsequent wave.

Value

A list of two objects: points and emulators

Examples

```
#excessive runtime
ranges <- list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05))
default <- full_wave(do.call('rbind.data.frame', SIRSample), ranges,
    SIREmulators$targets)
non_quad <- full_wave(do.call('rbind.data.frame', SIRSample), ranges,
    SIREmulators$targets, quadratic = FALSE)
second <- full_wave(SIRMultiWaveData[[2]], ranges, SIREmulators$targets,
    old_emulators = SIRMultiWaveEmulators[[1]])</pre>
```

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gamma_	_exp

Gamma-exponential correlation function

Description

For points x, xp, and a pair of hyperparameters gamma and theta, gives the gamma-exponential correlation between the two points.

Usage

```
gamma_exp(x, xp, hp)
```

Arguments

X	A data.frame of rows corresponding to position vectors
хр	A data.frame of rows corresponding to position vectors
hp	The hyperparameters theta (correlation length) and gamma (exponent), as a named list

Details

The gamma-exponential correlation function, for d = |x-x'|, is given by $\exp(-(d/\theta)^{\gamma})$. Gamma must be between 0 (exclusive) and 2 (inclusive).

Value

The gamma-exponential correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

```
gamma_exp(data.frame(a=1), data.frame(a=2), list(gamma = 1.5, theta = 0.1))
#> 1.846727e-14
gamma_exp(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(gamma = 1.3, theta = 0.2))
#> 0.0001399953
```

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Description

Given a set of trained emulators, this finds the next set of points that will be informative for a subsequent wave of emulation or, in the event that the current wave is the last desired, a set of points that optimally span the parameter region of interest. There are a number of different methods that can be utilised, alone or in combination with one another, to generate the points.

Usage

```
generate_new_design(
  ems,
  n_points,
  z,
  method = "default",
  cutoff = 3,
  plausible_set,
  verbose = interactive(),
  opts = NULL,
  ...
)
```

Arguments

ems A list of Emulator objects, trained on previous design points.

n_points The desired number of points to propose.

z The targets to match to.method Which methods to use.

cutoff The value of the cutoff to use to assess suitability.

plausible_set An optional set of known non-implausible points, to avoid LHD sampling.

verbose Should progress statements be printed to the console?

opts A named list of opts as described.

... Any parameters to pass via chaining to individual sampling functions (eg distro

for importance sampling or ordering for collecting emulators).

Details

If the method argument contains 'lhs', a Latin hypercube is generated and non-implausible points from this design are retained. If more points are accepted than the next design requires, then points are subselected using a maximin argument.

If method contains 'line', then line sampling is performed. Given an already established collection of non-implausible points, rays are drawn between pairs of points (selected so as to maximise

the distance between them) and more points are sampled along the rays. Points thus sampled are retained if they lie near a boundary of the non-implausible space, or on the boundary of the parameter region of interest.

If method contains 'importance', importance sampling is performed. Given a collection of non-implausible points, a mixture distribution of either multivariate normal or uniform ellipsoid proposals around the current non-implausible set are constructed. The optimal standard deviation (in the normal case) or radius (in the ellipsoid case) is determined using a burn-in phase, and points are proposed until the desired number of points have been found.

If method contains 'slice', then slice sampling is performed. Given a single known non-implausible point, a minimum enclosing hyperrectangle (perhaps after transforming the space) is determined and points are sampled for each dimension of the parameter space uniformly, shrinking the minimum enclosing hyperrectangle as appropriate. This method is akin to to a Gibbs sampler.

If method contains 'optical', then optical depth sampling is used. Given a set of non-implausible points, an approximation of the one-dimensional marginal distributions for each parameter can be determined. From these derived marginals, points are sampled and subject to rejection as in the LHD sampling.

For any sampling strategy, the parameters ems, n_points, and z must be provided. All methods rely on a means of assessing point suitability, which we refer to as an implausibility measure. By default, this uses nth-maximum implausibility as provided by nth_implausible; a user-defined method can be provided instead by supplying the function call to opts[["accept_measure"]]. Any such function must take at least five arguments: the emulators, the points, the targets, and a cutoff, as well as a . . . argument to ensure compatibility with the default behaviour of the point proposal method. Note that, in accordance with the default functionality of nth_implausible, if emulating more than 10 outputs and an explicit opts\$nth argument is not provided, then second-max implausibility is used as the measure.

The option opts[["seek"]] determines how many points should be chosen that have a higher probability of matching targets, as opposed to not missing targets. Due to the danger of such an approach, this value should not be too high and should be used sparingly at early waves; even at later waves, it is inadvisable to seek more than 10% of the output points using this metric. The default is seek = 0, and can be provided as either a percentage of points desired (in the range [0,1]) or the fixed number of points.

The default behaviour is as follows. A set of initial points are generated from a large LHD; line sampling is performed to find the boundaries of the space; then importance sampling is used to fill out the space. The proposed set of points are thinned and both line and importance sampling are applied again; this resampling behaviour is controlled by opts[["resample"]], where resample = n indicates that the proposal will be thinned and resampled from n times (resulting in n+1 proposal stages).

In regions where the non-implausible space at a given cutoff value is very hard to find, the point proposal will start at a higher cutoff where it can find a space-filling design. Given such a design at a higher cutoff, it can subselect to a lower cutoff by demanding some percentage of the proposed points are retained and repeat. This approach terminates if the 'ladder' of cutoffs reaches the desired cutoff, or if the process asymptotes at a particular higher cutoff. The opts ladder_tolerance and cutoff_tolerance determine the minimum improvement required in consecutive cutoffs for the process to not be considered to be asymptoting and the level of closeness to the desired cutoff at which we are prepared to stop, respectively. For instance, setting ladder_tolerance to 0.1 and cutoff_tolerance to 0.01, with a cutoff of 3, will terminate the process if two consecutive cutoffs

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proposed are within 0.1 of each other, or when the points proposed all have implausibility less than the 3.01.

These methods may work slowly, or not at all, if the target space is extremely small in comparison with the initial non-yet-ruled-out (NROY) space; it may also fail to give a representative sample if the target space is formed of disconnected regions of different volumes.

Value

A data frame containing the set of new points upon which to run the model.

Arguments within opts

accept_measure A custom implausibility measure to be used.

cluster Whether to try to apply emulator clustering.

cutoff_tolerance Tolerance for an obtained cutoff to be similar enough to that desired.

ladder_tolerance Tolerance with which to determine if the process is asymptoting.

nth The level of nth implausibility to apply, if using the default implausibility.

resample How many times to perform the resampling step once points are found.

seek How many 'good' points should be sought: either as an integer or a ratio.

to file If output is to be written to file periodically, the file location.

points.factor (LHS, Cluster LHS) How many more points than desired to sample.

pca lhs (LHS) Whether to apply PCA to the space before proposing.

n lines (Line) How many lines to draw.

ppl (Line) The number of points to sample per line.

imp_distro (**Importance**) The distribution to propose around points.

imp_scale (**Importance**) The radius, or standard deviation, of proposed distributions.

pca_slice (Slice) Whether to apply PCA to the space before slice sampling.

seek_distro (Seek) The distribution to apply when looking for 'good' points.

Examples

```
# Excessive runtime
# A simple example that uses number of the native and ... parameter opts.
pts <- generate_new_design(SIREmulators$ems, 100, SIREmulators$targets,
distro = 'sphere', opts = list(resample = 0))
# Non-default methods
pts_slice <- generate_new_design(SIREmulators$ems, 100, SIREmulators$targets,
method = 'slice')
## Example using custom measure functionality
custom_measure <- function(ems, x, z, cutoff, ...) {
imps_df <- nth_implausible(ems, x, z, get_raw = TRUE)
sorted_imps <- t(apply(imps_df, 1, sort, decreasing = TRUE))
imps1 <- sorted_imps[,1] <= cutoff
imps2 <- sorted_imps[,2] <= cutoff - 0.5
constraint <- apply(x, 1, function(y) y[[1]] <= 0.4)</pre>
```

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```
return(imps1 & imps2 & constraint)
}
pts_custom <- generate_new_design(SIREmulators$ems, 100, SIREmulators$targets,
opts = list(accept_measure = custom_measure))</pre>
```

generate_new_runs

Generate Proposal Points

Description

This function is deprecated in favour of generate_new_design.

Usage

```
generate_new_runs(
  ems,
  n_points,
  z,
  method = "default",
  cutoff = 3,
  plausible_set,
  verbose = interactive(),
  opts = NULL,
  ...
)
```

Arguments

ems A list of Emulator objects, trained on previous design points.

n_points The desired number of points to propose.

z The targets to match to.method Which methods to use.

cutoff The value of the cutoff to use to assess suitability.

plausible_set An optional set of known non-implausible points, to avoid LHD sampling.

verbose Should progress statements be printed to the console?

opts A named list of opts as described.

... Any parameters to pass via chaining to individual sampling functions (eg distro

for importance sampling or ordering for collecting emulators).

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Details

Given a set of trained emulators, this finds the next set of points that will be informative for a subsequent wave of emulation or, in the event that the current wave is the last desired, a set of points that optimally span the parameter region of interest. There are a number of different methods that can be utilised, alone or in combination with one another, to generate the points.

If the method argument contains 'lhs', a Latin hypercube is generated and non-implausible points from this design are retained. If more points are accepted than the next design requires, then points are subselected using a maximin argument.

If method contains 'line', then line sampling is performed. Given an already established collection of non-implausible points, rays are drawn between pairs of points (selected so as to maximise the distance between them) and more points are sampled along the rays. Points thus sampled are retained if they lie near a boundary of the non-implausible space, or on the boundary of the parameter region of interest.

If method contains 'importance', importance sampling is performed. Given a collection of non-implausible points, a mixture distribution of either multivariate normal or uniform ellipsoid proposals around the current non-implausible set are constructed. The optimal standard deviation (in the normal case) or radius (in the ellipsoid case) is determined using a burn-in phase, and points are proposed until the desired number of points have been found.

If method contains 'slice', then slice sampling is performed. Given a single known non-implausible point, a minimum enclosing hyperrectangle (perhaps after transforming the space) is determined and points are sampled for each dimension of the parameter space uniformly, shrinking the minimum enclosing hyperrectangle as appropriate. This method is akin to to a Gibbs sampler.

If method contains 'optical', then optical depth sampling is used. Given a set of non-implausible points, an approximation of the one-dimensional marginal distributions for each parameter can be determined. From these derived marginals, points are sampled and subject to rejection as in the LHD sampling.

For any sampling strategy, the parameters ems, n_points, and z must be provided. All methods rely on a means of assessing point suitability, which we refer to as an implausibility measure. By default, this uses nth-maximum implausibility as provided by nth_implausible; a user-defined method can be provided instead by supplying the function call to opts[["accept_measure"]]. Any such function must take at least five arguments: the emulators, the points, the targets, and a cutoff, as well as a . . . argument to ensure compatibility with the default behaviour of the point proposal method.

The option opts[["seek"]] determines how many points should be chosen that have a higher probability of matching targets, as opposed to not missing targets. Due to the danger of such an approach if a representative space-filling design over the space, this value should not be too high and should be used sparingly at early waves; even at later waves, it is inadvisable to seek more than 10% of the output points using this metric. The default is seek = 0, and can be provided as either a percentage of points desired (in the range [0,1]) or the fixed number of points.

The default behaviour is as follows. A set of initial points are generated from a large LHD; line sampling is performed to find the boundaries of the space; then importance sampling is used to fill out the space. The proposed set of points are thinned and both line and importance sampling are applied again; this resampling behaviour is controlled by opts[["resample"]], where resample = n indicates that the proposal will be thinned and resampled from n times (resulting in n+1 proposal stages).

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In regions where the non-implausible space at a given cutoff value is very hard to find, the point proposal will start at a higher cutoff where it can find a space-filling design. Given such a design at a higher cutoff, it can subselect to a lower cutoff by demanding some percentage of the proposed points are retained and repeat. This approach terminates if the 'ladder' of cutoffs reaches the desired cutoff, or if the process asymptotes at a particular higher cutoff. The opts ladder_tolerance and cutoff_tolerance determine the minimum improvement required in consecutive cutoffs for the process to not be considered to be asymptoting and the level of closeness to the desired cutoff at which we are prepared to stop, respectively. For instance, setting ladder_tolerance to 0.1 and cutoff_tolerance to 0.01, with a cutoff of 3, will terminate the process if two consecutive cutoffs proposed are within 0.1 of each other, or when the points proposed all have implausibility less than the 3.01.

These methods may work slowly, or not at all, if the target space is extremely small in comparison with the initial non-yet-ruled-out (NROY) space; it may also fail to give a representative sample if the target space is formed of disconnected regions of different volumes.

Value

A data frame containing the set of new points upon which to run the model.

Arguments within opts

accept_measure A custom implausibility measure to be used.

cluster Whether to try to apply emulator clustering.

cutoff_tolerance Tolerance for an obtained cutoff to be similar enough to that desired.

ladder_tolerance Tolerance with which to determine if the process is asymptoting.

nth The level of nth implausibility to apply, if using the default implausibility.

resample How many times to perform the resampling step once points are found.

seek How many 'good' points should be sought: either as an integer or a ratio.

to_file If output is to be written to file periodically, the file location.

points.factor (LHS, Cluster LHS) How many more points than desired to sample.

pca_lhs (LHS) Whether to apply PCA to the space before proposing.

n_lines (Line) How many lines to draw.

ppl (Line) The number of points to sample per line.

imp distro (Importance) The distribution to propose around points.

imp_scale (**Importance**) The radius, or standard deviation, of proposed distributions.

pca_slice (Slice) Whether to apply PCA to the space before slice sampling.

seek_distro (**Seek**) The distribution to apply when looking for 'good' points.

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get_diagnostic	Diagnostic Tests for Emulators	

Description

Given an emulator, return a diagnostic measure.

Usage

```
get_diagnostic(
  emulator,
  targets = NULL,
  validation = NULL,
  which_diag = "cd",
  stdev = 3,
  cleaned = NULL,
  warn = TRUE,
  kfold = NULL,
  ...
)
```

Arguments

emulator	An object of class Emulator
targets	If desired, the target values for the output(s) of the system
validation	If provided, the emulator is tested against the outputs of these points
which_diag	Which diagnostic measure to use (choosing from cd, ce, se above)
stdev	For 'cd', a measure of the allowed distance from prediction and reality
cleaned	Internal for stochastic emulators
warn	Should a warning be shown if ce is chosen and no targets provided?
kfold	Mainly internal: pre-computed k-fold diagnostic results for output
	Any other parameters to be passed through to subfunctions.

Details

An emulator's suitability can be checked in a number of ways. This function combines all current diagnostics available in the package, returning a context-dependent data.frame containing the results

Comparison Diagnostics (cd): Given a set of points, the emulator expectation and variance are calculated. This gives a predictive range for the input point, according to the emulator. We compare this against the actual value given by the simulator: points whose emulator prediction is further away from the simulator prediction are to be investigated. This 'distance' is given by stdev, and an emulator prediction correspondingly should not be further away from the simulator value than stdev*uncertainty.

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Classification Error (ce): Given a set of targets, the emulator can determine implausibility of a point with respect to the relevant target, accepting or rejecting it as appropriate. We can define a similar 'implausibility' function for the simulator: the combination of the two rejection schemes gives four classifications of points. Any point where the emulator would reject the point but the simulator would not should be investigated.

Standardized Error (se): The known value at a point, combined with the emulator expectation and uncertainty, can be combined to provide a standardized error for a point. This error should not be too large, in general. but the diagnostic is more useful when looking at a collection of such measures, where systematic bias or over/underconfidence can be seen.

Which of the diagnostics is performed can be controlled by the which_diag argument. If performing classification error diagnostics, a set of targets must be provided; for all diagnostics, a validation (or holdout) set can be provided. If no such set is given, then the emulator diagnostics are performed with respect to its training points, using k-fold cross-validation.

Value

A data frame consisting of the input points, output values, and diagnostic measures.

See Also

```
validation_diagnostics
```

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), individual_errors(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

Examples

```
# Use the simple SIR model via SIREmulators
get_diagnostic(SIREmulators$ems$nS, validation = SIRSample$validation)
# Classification error fails without the set of targets
get_diagnostic(SIREmulators$ems$nI, SIREmulators$targets, SIRSample$validation, 'ce')
# No validation set: k-fold cross-validation will be used.
get_diagnostic(SIREmulators$ems$nR, which_diag = 'se')
```

Hierarchical Emulator Hierarchical Bayes Linear Emulator

Description

Creates a univariate emulator with hierarchical structure.

This object does not differ extensively from the standard Emulator object, so most of the functionality will not be listed here: the main difference is that it allows for the variance structure of the emulator to be modified by a higher order object. The typical usage is to create a variance emulator, whose predictions inform the behaviour of a mean emulator with regard to a stochastic process.

Constructor

```
HierarchicalEmulator$new(basis_f, beta, u, ranges, ...)
```

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Arguments

For details of shared arguments, see Emulator.

s_diag The function that modifies the structure of the Bayes Linear adjustment.

samples A numeric vector that indicates how many replicates each of the training points has.

em_type Whether the emulator is emulating a mean surface or a variance surface.

Constructor Details

See Emulator: the constructor structure is the same save for the new arguments discussed above.

Accessor Methods

 $get_exp(x, samps = NULL)$ Similar in form to the normal Emulator method; the samps argument allows the estimation of summary statistics derived from multiple realisations.

 $get_cov(x, xp = NULL, full = FALSE, samps = NULL)$ Differences here are in line with those described in get_exp .

Object Methods

Identical to those of Emulator: the one internal difference is that adjust returns a HierarchicalEmulator rather than a standard one.

References

Goldstein & Vernon (2016), in preparation

Examples

```
h_em <- emulator_from_data(BirthDeath$training, c('Y'),
  list(lambda = c(0, 0.08), mu = c(0.04, 0.13)), emulator_type = "variance")
names(h_em) # c("expectation', 'variance')</pre>
```

hit_by_wave

Output Hit Summary

Description

Provides a summary of numbers of points that hit n outputs

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Usage

```
hit_by_wave(
  waves,
  targets,
  input_names,
  measure = "mean",
  plt = FALSE,
  as.per = TRUE,
  grid.plot = TRUE,
  n.sig = 3
)
```

Arguments

waves The collection of waves, as a list of data.frames

targets The output targets

input_names The names of the input parameters

measure If stochastic, the measure to use to compare (see description)

plt If TRUE, results are plotted; else a data.frame is returned

as.per Should the data be percentages, or raw numbers?

grid.plot If plt = TRUE, determines the type of plot.

n.sig If (val, sigma) targets provided, how many sigma away from the mean do we

consider a match?

Details

Given a collection of wave points and the targets used in history matching, it might be informative to consider the proportion of points whose model output matches a given number of targets. This function provides by-wave information about how many parameter sets are matches to 0,1,2,...,n outputs.

The results of the analysis can be presented as a data. frame object where each row is a wave and each column a number of outputs; if plt = TRUE the results are instead presented visually, as a grid coloured by proportion of total points (if grid.plot = TRUE, the default) or as a series of discrete density lines, one per wave. The as.per argument determines whether the output values are raw or if they are calculated percentages of the total number of parameter sets for a given wave.

When the data arise from a stochastic model, and therefore parameter sets have multiple realisations, there are multiple ways to analyze the data (determined by measure). The options are "mean" to compare the means of realisations to the outputs; "real" to compare all individual realisations; and "stoch" to consider an output matched to if the mean lies within 3 standard deviations of the output, where the standard deviation is calculated over the realisations.

Value

Either a data.frame of results or a ggplot object plot

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See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
# Default Usage
hit_by_wave(SIRMultiWaveData, SIREmulators$targets, c('aSI', 'aIR', 'aSR'))
# Plotting - line plot or raw figures
hit_by_wave(SIRMultiWaveData, SIREmulators$targets, c('aSI', 'aIR', 'aSR'),
plt = TRUE, as.per = FALSE, grid.plot = FALSE)
```

idemc

IDEMC Point Generation

Description

Performs Implausibility-driven Evolutionary Monte Carlo

Usage

```
idemc(
  ems,
 Ν,
  targets,
  cutoff = 3,
  s = max(500, ceiling(N/5)),
  sn = s,
  p = 0.4
  thin = 1,
  pm = 0.9,
 w = 0.8,
 M = 10,
  detailed = FALSE,
  verbose = interactive(),
  get_burnt = FALSE,
  burnt = NULL
)
```

Arguments

ems The emulators to evaluate implausibility on

N The desired number of final points to generate
targets The target values for the emulated outputs

cutoff The desired implausibility cutoff of the final proposal

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s The number of points to generate at intermediate burn-in steps
sn The number of points to generate at the final burn-in stage
p The proportion of space that should remain between ladder rungs

thin The thinning factor: a factor T means N*T points are generated to obtain N

The probability that an idemc step will use mutation moves

The probability of local random walks in the mutation step

The number of mutations to perform in an IDEMC step

detailed If TRUE, points proposed at every rung will be returned

verbose Should information about burn-in be displayed during the process?

get_burnt If TRUE, the procedure stops after burn-in, returning seeding for a full IDEMC

proposal

burnt If provided, this is assumed to be the result of a burn-in (or a priori analysis)

Details

This method for generating points is focused on finding non-implausible regions that are either extremely small relative to the initial parameter domain, or have interesting structure (particularly disconnected structure) that would potentially be overlooked by more standard point generation methods. The method is robust but computationally intensive, compared to normal methods, and should not be used as a default - see generate_new_design for less computationally expensive methods.

The IDEMC method operates on an 'implausibility ladder', in the vein of common annealing methods. Each rung of the ladder is characterised by the implausibility threshold, and determinations are made about the structure of the points in each rung using clustering. One step of the evolutionary algorithm can consist of the following steps:

Mutation. A point is modified using a random-walk proposal, which can be a global move or a within-cluster move. Within-cluster moves are chosen with probability w. The move is retained if the new point satisfies the implausibility constraints of the rung.

Crossover. Points are re-organised in descending order of how active each variable is for the emulated outputs, and two different rungs are selected randomly. The points are 'mixed' using one-point real crossover at a random crossover point, producing two new points. The move is retained if both new points satisfy the relevant implausibility constraints of their rung.

Exchange. Two adjacent rungs are chosen and their points are swapped. The move is retained if the higher-implausibility rung is appropriate for being in the lower implausibility rung.

At a given step, one of mutation or crossover is performed, with probability of mutation being chosen determined by pm. If mutation is chosen, then M mutation moves are performed; else (n+1)/2 crossover moves are performed on the n rungs. Exchange is always performed and n+1 such moves are performed.

The choice of 'implausibility ladder' and clusters can be determined a priori, or else this function performs a burn-in phase to determine them. Points are generated using the idemc steps at the current rungs, and the next ladder rung implausibility is chosen by requiring that a proportion p of points from the previous rung are accepted in the new one. At each stage, s idemc steps are performed. Once the final rung has implausibility no larger than the desired cutoff, a final set of sn idemc steps are performed across all rungs to determine final clusters.

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Value

Either a list of data.frames, one per rung, or a single data.frame of points.

References

```
Vernon & Williamson (2013) doi:10.48550/arXiv.1309.3520
```

See Also

```
generate_new_design for more standard point generation methods
```

Examples

```
# Excessive runtime
idemc_res <- idemc(SIREmulators$ems, 200, SIREmulators$targets, s = 100, p = 0.3)</pre>
```

individual_errors

Predictive Error Plots

Description

Plots the predictive error with respect to a variety of quantities.

Usage

```
individual_errors(
  em,
  validation,
  errtype = "normal",
  xtype = "index",
  plottype = "normal"
)
```

Arguments

em The emulator to perform diagnostics on validation The validation set of points with output(s) errtype The type of individual error to be plotted. xtype The value to plot against plottype Whether to plot a standard or Q-Q plot.

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Details

The choice of errors to plot is controlled by errtype, and can be one of four things: normal, corresponding to the regular standardised errors; eigen, corresponding to the errors after reordering given by the eigendecomposition of the emulator covariance matrix; chol, similarly deriving errors after Cholesky decomposition; and cholpivot, deriving the errors after pivoted Cholesky decomposition.

What the errors are plotted with respect to is controlled by xtype. The options are index, which plots them in their order in the validation set; em, which plots errors with respect to the emulator prediction at that point; and any named parameter of the model, which plots with respect to the values of that parameter.

Finally, the plot type is controlled by plottype: this can be one of normal, which plots the errors; or qq, which produces a Q-Q plot of the errors.

The default output is to plot the standardised errors (with no decomposition) against the ordering in the validation set; i.e. errtype = "normal", xtype = "index", plottype = "normal".

Some combinations are not permitted, as the output would not be meaningful. Errors arising from an eigendecomposition cannot be plotted against either emulator prediction or a particular parameter (due to the transformation induced by the eigendecomposition); Q-Q plots are not plotted for a non-decomposed set of errors, as the correlation between errors makes it much harder to interpret.

Value

The relevant plot.

References

```
Bastos & O'Hagan (2009) <doi:10.1198/TECH.2009.08019>
```

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), residual_diag(), standard_errors(), summary_diag(), validation_diagnostics()
```

Examples

```
i1 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation)
i2 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation, "chol", "em")
i3 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation, "eigen", plottype = "qq")
i4 <- individual_errors(SIREmulators$ems$nS, SIRSample$validation, "cholpivot", xtype = "aSI")</pre>
```

matern

Matern correlation function

Description

For points x, xp, and a pair of hyperparameters nu and theta, gives the Matern correlation between the two points.

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Usage

```
matern(x, xp, hp)
```

Arguments

x A data.frame of rows corresponding to position vectors
 xp A data.frame of rows corresponding to position vectors
 hp The hyperparameters nu (smoothness) and theta (correlation length), as a named list

Details

At present, only half-integer arguments for nu are supported.

Value

The Matern correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

nth_implausible

nth Maximum Implausibility

Description

Computes the nth-maximum implausibility of points relative to a set of emulators.

Usage

```
nth_implausible(
  ems,
  x,
  z,
  n = NULL,
  max_imp = Inf,
  cutoff = NULL,
  sequential = FALSE,
  get_raw = FALSE,
```

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```
ordered = FALSE,
...
)
```

Arguments

ems	A set of Emulator objects or nested sets thereof (see description)
x	An input point, or data. frame of points.
z	The target values, in the usual form or nested thereof.
n	The implausibility level to return.
max_imp	A maximum implausibility to return (often used with plotting)
cutoff	A numeric value, or vector of such, representing allowed implausibility
sequential	Should the emulators be evaluated sequentially?
get_raw	Boolean - determines whether nth-implausibility should be applied.
ordered	If FALSE, emulators are ordered according to restrictiveness.
•••	Any additional arguments to pass to chained functions (e.g. ordering to pass to collect_emulators)

Details

For a collection of emulators, we often combine the implausibility measures for a given set of observations. The maximum implausibility of a point, given a set of univariate emulators and an associated collection of target values, is the largest implausibility of the collected set of implausibilities. The 2nd maximum is the maximum of the set without the largest value, and so on. By default, maximum implausibility will be considered when there are fewer than 10 targets to match to; otherwise second-maximum implausibility is considered.

If sequential = TRUE and a specific cutoff has been provided, then the emulators' implausibility will be evaluated one emulator at a time. If a point is judged implausible by more than n emulators, FALSE is returned without evaluating any more. Due to R efficiencies, this is more efficient than the 'evaluate all' method once more than around 10 emulators are considered.

This function also deals with variance emulators and bimodal emulators, working in a nested fashion. If targets are provided for both the expectation and variance as a list, then given ems = list(expectation = ..., variance = ...) the implausibility is calculated with respect to both sets of emulators, maximising as relevant. If targets are provided in the 'normal' fashion, then only the mean emulators are used. The bimodal case is similar; given a set of emulators list(mode1 = list(expectation = ..., variance = ...), ...) then each mode has implausibility evaluated separately. The results from the two modes are combined via piecewise minimisation.

Value

Either the nth maximum implausibilities, or booleans (if cutoff is given).

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Examples

```
# A single point
nth_implausible(SIREmulators$ems, data.frame(aSI = 0.4, aIR = 0.25, aSR = 0.025),
SIREmulators$targets)
# A data.frame of points
grid <- expand.grid(</pre>
 aSI = seq(0.1, 0.8, length.out = 4),
 aIR = seq(0, 0.5, length.out = 4),
 aSR = seq(0, 0.05, length.out = 4)
# Vector of numerics
i1 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets)</pre>
# Vector of booleans (same as i1 <= 3)</pre>
i2 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets, cutoff = 3)</pre>
# Throws a warning as n > no. of targets
i3 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets, n = 4)</pre>
# Vector of booleans (note different output to i2)
i4 <- nth_implausible(SIREmulators$ems, grid, SIREmulators$targets,</pre>
 cutoff = c(4, 2.5, 2))
# Variance Emulators
v_ems <- emulator_from_data(BirthDeath$training, c('Y'),</pre>
list(lambda = c(0, 0.08), mu = c(0.04, 0.13)), emulator_type = "variance")
v_{targs} = list(expectation = list(Y = c(90, 110)), variance = list(Y = c(55, 95)))
nth_implausible(v_ems, unique(BirthDeath$validation[,1:2]), v_targs)
## If there is a mismatch between emulators and targets, expectation is assumed
nth_implausible(v_ems$expectation, unique(BirthDeath$validation[,1:2]), v_targs)
nth_implausible(v_ems, unique(BirthDeath$validation[,1:2]), v_targs$expectation)
```

orn_uhl

Ornstein-Uhlenbeck correlation function

Description

For points x, xp, and a hyperparameter theta, gives the Ornstein-Uhlenbeck correlation between the two points.

Usage

```
orn_uhl(x, xp, hp)
```

Arguments

Χ	A data frame of rows corresponding to position vectors
хр	A data frame of rows corresponding to position vectors
hp	The hyperparameter theta (correlation length) in a named list

48 output_plot

Details

This correlation function can be seen as a specific case of the Matern correlation function when nu = 1/2.

Value

The Ornstein-Uhlenbeck correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

```
orn_uhl(data.frame(a=1), data.frame(a=2), list(theta = 0.1))
#> 4.539993e-05
orn_uhl(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(theta = 0.2))
#> 0.00469197
orn_uhl(data.frame(a=1,b=1,c=1), data.frame(a=1.2,b=0.9,c=0.6), list(theta = 0.2)) ==
matern(data.frame(a=1,b=1,c=1), data.frame(a=1.2,b=0.9,c=0.6), list(theta = 0.2, nu = 0.5)) #> TRUE
```

output_plot

Emulator Expectation Against Target Outputs

Description

Plots emulator expectation across the parameter space, with comparison to the corresponding target values (with appropriate uncertainty).

Usage

```
output_plot(ems, targets, points = NULL, npoints = 1000)
```

Arguments

ems The Emulator objects.

targets A named list of observations, given in the usual form.

Points A list of points at which the emulators should be evaluated.

npoints If no points are provided, the number of input points to evaluate at.

Details

If a points data.frame is not provided, then points are sampled uniformly from the input region. Otherwise, the provided points are used: for example, if a representative sample of the current NROY space is available.

plot_actives 49

Value

A ggplot object

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
output_plot(SIREmulators$ems, SIREmulators$targets)
output_plot(SIREmulators$ems, SIREmulators$targets, points = SIRSample$training)
```

plot_actives

Active variable plotting

Description

For a set of emulators, demonstrate which variables are active.

Usage

```
plot_actives(ems, output_names = NULL, input_names = NULL)
```

Arguments

ems The list of emulators to consider

output_names The names of the outputs to include in the plot, if not all input_names The names of the inputs to include in the plot, if not all

Details

Each emulator has a list of 'active' variables; those which contribute in an appreciable way to its regression surface. It can be instructive to examine the differences in active variables for a give collection of emulators. The plot here produces an nxp grid for n emulators in p inputs; a square is blacked out if that variable does not contribute to that output.

Both the outputs and inputs can be restricted to collections of interest, if desired, with the optional output_names and input_names parameters.

Value

A ggplot object corresponding to the plot

50 plot_lattice

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
plot_actives(SIREmulators$ems)
# Remove the nR output and aIR input from the plot
plot_actives(SIREmulators$ems, c('nS', 'nI'), c('aSI', 'aSR'))
# Note that we can equally restrict the emulator list...
plot_actives(SIREmulators$ems[c('nS', 'nI')], input_names = c('aSI', 'aSR'))
```

plot_lattice

Plot Lattice of Emulator Implausibilities

Description

Plots a set of projections of the full-dimensional input space.

Usage

```
plot_lattice(
  ems,
  targets,
  ppd = 20,
  cb = FALSE,
  cutoff = 3,
  maxpoints = 50000,
  imp_breaks = NULL,
  contour = TRUE,
  ranges = NULL,
  raster_imp = FALSE,
  plot_vars = NULL,
  fixed_vars = NULL
)
```

Arguments

ems The Emulator objects in question.

targets The corresponding target values.

ppd The number of points to sample per dimension.

cb Whether or not a colourblind-friendly plot should be produced.

cutoff The cutoff value for non-implausible points.

maxpoints The limit on the number of points to be evaluated.

plot_lattice 51

imp_breaks If plotting nth maximum implausibility, defines the levels at which to draw con-

tours.

contour Logical determining whether to plot implausibility contours or not.

ranges Parameter ranges. If not supplied, defaults to emulator ranges.

raster_imp Should the implausibility plots be rasterised?

plot_vars If provided, indicates which subset of parameters to plot.

fixed_vars If provided, indicates the fixed value of the plot-excluded parameters.

Details

The plots are:

One dimensional optical depth plots (diagonal);

Two dimensional optical depth plots (lower triangle);

Two dimensional minimum implausibility plots (upper triangle).

The optical depth is calculated as follows. A set of points is constructed across the full d-dimensional parameter space, and implausibility is calculated at each point. The points are collected into groups based on their placement in a projection to a one- or two-dimensional slice of the parameter space. For each group, the proportion of non-implausible points is calculated, and this value in [0,1] is plotted. The minimum implausibility plots are similar, but with minimum implausibility calculated rather than proportion of non-implausible points.

The maxpoints argument is used as a cutoff for if a regular ppd grid would result in a very large number of points. If this is the case, then maxpoints points are sampled uniformly from the region instead of regularly spacing them.

If only a subset of parameters are relevant, then the plot_vars and fixed_vars can be used to specify the subset. If plot_vars is provided, corresponding to a list of parameter names, then those parameters not included are fixed to their mid-range values; if fixed_vars is provided as a named list, then the parameters not included are fixed to the corresponding specified values.

Value

A ggplot object

References

Bower, Goldstein & Vernon (2010) <doi:10.1214/10-BA524>

See Also

Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()

52 plot_wrap

Examples

```
# Excessive runtime
plot_lattice(SIREmulators$ems, SIREmulators$targets, ppd = 10)
plot_lattice(SIREmulators$ems$nS, SIREmulators$targets)
plot_lattice(SIREmulators$ems, SIREmulators$targets, plot_vars = c('aSI', 'aIR'))
plot_lattice(SIREmulators$ems, SIREmulators$targets, fixed_vars = list(aSR = 0.03))
```

plot_wrap

Plot proposed points

Description

A wrapper around R's base plot to show proposed points

Usage

```
plot_wrap(points, ranges = NULL, p_size = 0.5)
```

Arguments

points The points to plot ranges The parameter ranges

p_size The size of the plotted points (passed to cex)

Details

Given a set of points proposed from emulators at a given wave, it's often useful to look at how they are spread and where in parameter space they tend to lie relative to the original ranges of the parameters. This function provides pairs plots of the parameters, with the bounds of the plots calculated with respect to the parameter ranges provided.

Value

The corresponding pairs plot

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
plot_wrap(SIRSample$training[,1:3], SIREmulators$ems[[1]]$ranges)
```

problem_data 53

problem_data

Data for an interesting emulation problem

Description

An RData object consisting of four objects: a data.frame data of 208 points, a set targets of 19 targets for outputs, a set ranges of 21 ranges for inputs, and a data.frame extra of 26 additional points. This dataset is used to demonstrate some of the subtleties of emulation in the vignettes, where data transformations can be useful and careful attention should be paid to emulation at early waves.

Usage

problem_data

Format

A list of objects:

data The training data of 'space-filling' runs

targets The output targets to match to

ranges The input ranges over which the system is valid

extra A set of 'extra' points, generated around a known point of best fit.

Proto_emulator

Prototype Class for Emulator-like Objects

Description

Converts a prediction object into a form useable in hmer.

The history matching process can be used for objects that are not created by the hmer package: most notably Gaussian Process (GP) emulators but even for simple linear models. This R6 class converts such an object into a form that can be called directly and reliably by the methods of the package, including for visualisation and diagnostics.

Constructor

```
Proto_emulator$new(ranges, output_name, predict_func, variance_func, ...)
```

54 Proto_emulator

Arguments

Required:

ranges A list of ranges for the inputs to the model.

output_name The name of the output modelled.

predict_func The function that provides the predictions at a new point or points. The first argument of this function should be x, where x is a data.frame of points. Additional arguments can be specified as long as they match additional objects passed via . . . (see below for details).

variance_func The function that encodes the prediction error due to the model of choice. This, too, takes an argument x as above as its first argument. Additional arguments can be specified as long as they match additional objects passed via . . . (see below for details).

Optional:

implausibility_func A function that takes points x and a target z (and potentially a cutoff value cutoff and additional arguments) and returns a measure of closeness of the predicted value to the target (or a boolean representing whether the prediction is within the specified cutoff). Any custom implausibility should satisfy the definition: that is, a point that is unlikely to match to the observation should have higher implausibility than a point likely to match to the observation. If, for example, a likelihood to be maximised is used as a surrogate for an implausibility function, then one should transform it accordingly.

If this argument is not provided, the standard implausibility is used: namely, the absolute value of the difference between prediction and observation, divided by the square root of the sum in quadrature of the errors.

Additional arguments can be specified as long as they match additional objects passed via ... (see below for details).

print_func If the prediction object has a suitable print function that one wishes to transfer to the R6 class (e.g. summary.lm), it is specified here.

... Additional objects to pass to predict_func, variance_func, implausibility_func or print_func. The names of these objects must match the additional argument names in the corresponding functions.

Constructor Details

The constructor must take, as a minimum, the first four arguments (input ranges, output name, and the prediction and variance functions). Default behaviour exists if the implausibility function and print function are not specified. The output of the constructor is an R6 object with the classes "Emulator" and "EmProto".

Accessor Methods

Note that these have the same external structure as those in Emulator.

get_exp(x) Returns the prediction.

get_cov(x) Returns the prediction error.

implausibility(x, z, cutoff = NULL) Returns the 'implausibility'.

print() Prints relevant details of the object.

rat_quad 55

Examples

```
# Use linear regression with an "error" on the SIR dataset.
ranges <- list(aSI = c(0.1, 0.8), aIR = c(0, 0.5), aSR = c(0, 0.05))
targets <- SIREmulators$targets</pre>
lms <- purrr::map(names(targets),</pre>
 ~step(lm(data = SIRSample$training[,c(names(ranges), .)],
  formula = as.formula(paste0(., "~(",
   paste0(names(ranges), collapse = "+"),
   ")^2"
 ))
), trace = 0)
# Set up the proto emulators
proto_ems <- purrr::map(seq_along(lms), function(l) {</pre>
  Proto_emulator$new(
     ranges,
     names(targets)[1],
     function(x) predict(lms[[1]], x),
     function(x) predict(lms[[1]], x, se.fit = TRUE)$se.fit^2 +
        predict(lms[[1]], x, se.fit = TRUE)$residual.scale^2,
     print_func = function() print(summary(lms[[1]]))
}) |> setNames(names(targets))
# Test with some hmer functions
nth_implausible(proto_ems, SIRSample$validation, targets)
emulator_plot(proto_ems)
emulator_plot(proto_ems, 'imp', targets = targets)
validation_diagnostics(proto_ems, targets, SIRSample$validation)
new_points <- generate_new_design(proto_ems, 100, targets)</pre>
```

rat_quad

Rational Quadratic correlation function

Description

For points x, xp, and a pair of hyperparameters alpha and theta, gives the rational quadratic correlation between the two points.

Usage

```
rat_quad(x, xp, hp)
```

Arguments

x A data.frame of rows corresponding to position vectors

xp A data.frame of rows corresponding to position vectors

hp The hyperparameters alpha (exponent and scale) and theta (correlation length)

56 residual_diag

Details

This correlation function, for d = |x-x'|, has the form $(1 + d^2/(2\alpha\theta^2))^{-\alpha}$, and can be seen as a superposition of exponential-squared correlation functions.

Value

The rational quadratic correlation between x and xp.

References

Rasmussen & Williams (2005) <ISBN: 9780262182539>

Examples

```
rat_quad(data.frame(a=1), data.frame(a=2), list(alpha = 1.5, theta = 0.1))
#> 0.004970797
rat_quad(data.frame(a=1,b=2,c=-1),data.frame(a=1.5,b=2.9,c=-0.7), list(alpha = 1.5, theta = 0.2))
#> 0.02904466
```

residual_diag

Emulator Regression Residuals

Description

Plots the emulator residuals.

Usage

```
residual_diag(emulator, histogram = FALSE, ...)
```

Arguments

emulator The emulator to consider.

histogram Should a histogram or a scatter plot be shown? Default: FALSE

... Any additional arguments (used internally)

Details

An emulator is composed of two parts: a global regression surface, and a local correlation structure. It can sometimes be informative to examine the residuals of the regression surface on the training set, to determine the extent to which the regression surface is being 'corrected for' by the correlation structure.

Value

A set of residuals, standardised by the regression surface residual standard error.

simulator_plot 57

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), standard_errors(), summary_diag(), validation_diagnostics()
```

Examples

```
residual_diag(SIREmulators$ems$nS)
residual_diag(SIREmulators$ems$nI, TRUE)
```

simulator_plot

Plot simulator outputs for multiple waves

Description

Plots the simulator results for points at successive waves.

Usage

```
simulator_plot(
  wave_points,
  z,
  zero_in = TRUE,
  palette = NULL,
  wave_numbers = seq(ifelse(zero_in, 0, 1), length(wave_points) - ifelse(zero_in, 1, 0)),
  normalize = FALSE,
  logscale = FALSE,
  byhit = FALSE,
  barcol = "#4444444",
  ...
)
```

Arguments

wave_points The set of wave points, as a list of data.frames The set of target values for each output z zero_in Is wave zero included? Default: TRUE palette If a larger palette is required, it should be supplied here. Which waves to plot. If not supplied, all waves are plotted. wave_numbers normalize If true, plotting is done with rescaled target bounds. logscale If true, targets are log-scaled before plotting. byhit Should runs be grouped by number of targets hit, rather than wave? The colour of the target error bars/bounds barcol Optional parameters (not to be used directly)

58 SIREmulators

Details

The values plotted are the outputs from the simulator; the points passed to it are the points suggested by that wave of emulators. By default, wave 0 is included. A colour scheme is chosen outright for all invocations of this function: it is a 10-colour palette. If more waves are required, then an alternative palette should be selected.

The output can be plotted in a number of ways: raw; with outputs transformed to log scale; or with targets normalised so that target bounds are all [-1, 1]. These two options may be helpful in visualising behaviour when outputs have vastly different scales, but one still wishes to see them all in the same plot: these options can be toggled by setting logscale = TRUE or normalize = TRUE respectively. The data can be grouped in two ways, either colouring by wave of emulation (default) or by the number of targets hit; the latter option is enabled by setting byhit = TRUE.

Value

A ggplot object.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), space_removed(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
simulator_plot(SIRMultiWaveData, SIREmulators$targets)
simulator_plot(SIRMultiWaveData[2:4], SIREmulators$targets,
zero_in = FALSE, wave_numbers = c(1,3))
simulator_plot(SIRMultiWaveData, SIREmulators$targets, byhit = TRUE)
```

SIREmulators

Sample Emulators

Description

An RData object containing three trained emulators, and the associated targets, for the SIR example. The emulators have been trained on the SIRSample training dataset using methods documented in this package.

Usage

SIREmulators

Format

A list containing two objects:

```
ems The trained Emulator objects.
```

targets The targets to match to, as a named list.

SIRImplausibility 59

SIRImplausibility

Sample Implausibility Data

Description

A dataset containing 1000 points from the region bounded by [0.1, 0.8], [0, 0.5], [0, 0.05] for aSI, aIR and aSR respectively. Implausibility has been calculated (for emulators trained on the SIRSample training dataset) for each of the outputs nS, nI, nR, and the maximum implausibility is included. The target values used in calculating implausibility were:

```
nS between 324 and 358nI mean 143 (sigma 7.15)nR between 490 and 542
```

Usage

SIRImplausibility

Format

A data frame with 1000 rows and 7 variables:

aSI Infection: transition rate from S to I **aIR** Recovery: transition rate from I to R

aSR Waning Immunity: transition rate from R to S

nS Implausibility for nS**nI** Implausibility for nI

nR Implausibility for nRI Maximum implausibility

SIRMultiWaveData

Sample Multi-wave Results

Description

An rda object containing four data.frames: an initial set of points also provided in SIRSample, and the 90 points generated at each of three subsequent waves. The trained emulators are provided in SIRMultiWaveEmulators.

Usage

SIRMultiWaveData

60 SIRSample

Format

A list of data.frame objects:

Wave 0 The initial points used in other examples

Wave 1 Points generated from the wave 1 emulators

Wave 2 Points generated from the wave 2 emulators

Wave 3 Points generated from the wave 3 emulators

SIRMultiWaveEmulators Sample Multi-wave Emulators

Description

An rda object containing three waves of emulators applied to SIR model (described in SIRSample). The corresponding points (both training and validation) are stored in SIRMultiWaveData.

Usage

SIRMultiWaveEmulators

Format

A list containing Emulator objects:

Wave 1 Emulators trained on Wave 0, generating wave 1 points

Wave 2 Emulators trained on the results of the above wave 1 points

Wave 3 Emulators trained on the results of the above wave 2 points

SIRSample

Sample SIR data

Description

A small dataset containing points generated from a simple deterministic SIR model. The model contains three input parameters, and generates three output parameters. The initial populations are 950 susceptible (S), 50 infected (I), and 0 recovered (R). The final values are taken at time t=10.

Usage

SIRSample

SIR_stochastic 61

Format

A list of two data frames. The first has 30 rows and 6 variables, the second 60 rows and 6 variables. The structure is the same in both cases:

aSI Infection: transition rate from S to I
aIR Recovery: transition rate from I to R
aSR Waning immunity: transition rate from R to S
nS Final number of S
nI Final number of I

Details

The model operates using simple differential equations, where

S' = aSR*R - aSI*S*I/(S+I+R) I' = aSI*S*I/(S+I+R) - aIR*I R' = aIR*I - aSR*R.

nR Final number of R

SIR_stochastic

Stochastic SIR Data

Description

An RData object consisting of two data.frames (in a similar fashion to BirthDeath). The first consists of 30 points in the parameter space (aSI, aIR, aSR), each of which has been inputted into the Gillespie algorithm for the stochastic version of the model used in GillespieSIR (but with changed starting conditions) 100 times. The second has similar form but for 20 unique points, each with 100 repetitions.

Usage

SIR_stochastic

Format

A list of two data frames training and validation: each has the following columns:

aSI Infection rate

aIR Recovery rate

aSR Waning immunity rate

I10 (25, 50) The number of infected people at t = 10 (25, 50)

R10 (25, 50) The number of recovered people at t = 10 (25, 50)

62 space_removal

Details

The outputs observed are the numbers of infected (I) and recovered (R) people at time points t = 10, 25, 50. All outputs display some level of bimodality. The initial conditions to generate the runs had S(0)=995, I(0)=5, R(0)=0.

space_removal

Percentage of Space Removed

Description

For a wave of emulators, estimates the proportion of space removed at this wave.

Usage

```
space_removal(
  ems,
  targets,
  points = NULL,
  ppd = NULL,
  cutoff = 3,
  individual = TRUE
)
```

Arguments

ems The emulators to compute over, as a list

targets The output target values points The points to test against

ppd If no points are provided and uniform grid is wanted, the number of points per

parameter dimension.

cutoff The cutoff value for implausibility

individual If true, gives emulator-by-emulator results; otherwise works with maximum im-

plausibility

Details

Given a collection of emulators corresponding to a wave, we can look at an estimate of the proportion of points from previous waves that will be accepted at this wave, either on an emulator-by-emulator basis (to see which outputs are most restrictive) or as an all-wave determination.

Naturally, such a statement will be an estimate of the restriction on the full space (which will become more unreliable for higher dimensions), but it can give an order-of-magnitude statement, or useful comparators between different emulators in a wave.

If no points are provided, the training points for the emulators are used. For best results, a good number of points should be given: typically one should consider using as many points as one knows to be in the NROY space (including any validation points, if accessible).

space_removed 63

Value

A numeric corresponding to the proportions of points removed.

See Also

space_removed for a visualisation of the space removal.

Examples

```
space_removal(SIREmulators$ems, SIREmulators$targets,
  rbind(SIRSample$training, SIRSample$validation))
space_removal(SIREmulators$ems, SIREmulators$targets,
  rbind(SIRSample$training, SIRSample$validation), individual = FALSE)
```

space_removed

Space Removal Diagnostics

Description

Finds the proportion of space removed as a function of implausibility cut-off and of one of structural discrepancy, emulator variance, or correlation hyperparameter(s).

Usage

```
space_removed(
   ems,
   targets,
   ppd = 10,
   u_mod = seq(0.8, 1.2, by = 0.1),
   intervals = seq(0, 10, length.out = 200),
   modified = "obs",
   maxpoints = 50000
)
```

Arguments

ems	The Emulator objects.
targets	The corresponding targets to match to.
ppd	The number of points per input dimension to sample at.
u_mod	The proportional values by which to inflate/deflate the relevant statistic.
intervals	The interval values of the implausibility cutoff at which to evaluate.
modified	The statistic to modify: obs, disc, var or hp (see above)
maxpoints	The maximum number of points to evaluate at

64 standard_errors

Details

The reduction in space is found by evaluating a p^d regular grid, where p is chosen by ppd and d is the dimension of the input space. Larger values of p will give a more accurate reflection of the space removed, at a corresponding computational cost. For the purpose of quick-and-dirty diagnostics, ppd = 5 is sufficient: the default is 10.

The parameter modified can be one of three strings: 'obs' corresponding to observation uncertainty; 'disc' corresponding to internal and external discrepancy (as given in Emulator\$disc); 'var' corresponding to global emulator variance (as given by Emulator\$u_sigma), and 'hp' corresponding to the hyperparameters of the emulator correlation structure. In the first case, the implausibilities are recalculated for each inflation value; in the other two cases the emulators are retrained. For this reason, the 'var' and 'hp' options are computationally more intensive. The default is 'obs'.

The inflationary/deflationary values are chosen by u_mod: the default is to take 80%, 90%, 100%, 110%, and 120% of the original value as the variation. The proportion of points deemed non-implausible is checked at a set of implausibility cutoffs defined by intervals, and a plot is returned showing the relevant data.

Value

A ggplot object

See Also

space_removal for a numeric representation of space removed.

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), validation_pairs(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
space_removed(SIREmulators$ems, SIREmulators$targets, ppd = 5)
space_removed(SIREmulators$ems$nS, SIREmulators$targets,
ppd = 5, u_mod = seq(0.75, 1.25, by = 0.25), intervals = seq(2, 6, by = 0.1))
```

standard_errors

Standardized Error Diagnostics

Description

Shorthand function for diagnostic test 'se'.

subset_emulators 65

Usage

```
standard_errors(
  emulator,
  targets = NULL,
  validation = NULL,
  plt = interactive()
)
```

Arguments

emulator The emulator in question
targets The output targets
validation The validation set
plt Whether to plot or not

Details

For details of the function, see get_diagnostic and for the plot see analyze_diagnostic.

Value

A data.frame of failed points

References

Jackson (2018) http://etheses.dur.ac.uk/12826>

See Also

```
get_diagnostic, analyze_diagnostic, validation_diagnostics
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(),
get_diagnostic(), individual_errors(), residual_diag(), summary_diag(), validation_diagnostics()
```

subset_emulators

Subsetting for Bimodal/Variance Emulators

Description

Takes a collection of bimodal or stochastic emulators and subsets by output name.

Usage

```
subset_emulators(emulators, output_names)
```

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Arguments

emulators A set of emulators, often in nested form output_names The names of the desired outputs

Details

It can be useful to consider only a subset of outputs. In the normal case, this can be easily achieved; however, when the emulators are in a nested structure such as that provided by emulator_from_data with emulator_type = 'variance' or 'bimodal', it can be more involved. This function allows the easy selecting of emulators by name, returning a subset of them in the same form as the original object.

This function is compatible with 'standard' emulators; that is, those in a simple list, equivalent to subsetting over the collection of output names of the emulators that exist in output_names.

Value

An object of the same form as 'emulators'.

summary_diag Summary Statistics for Emulators

Januar y_arab

Description

Generates measures for emulator quality

Usage

```
summary_diag(emulator, validation, verbose = interactive())
```

Arguments

emulator The emulator to test

validation The validation set, consisting of points and output(s)

verbose Should statistics be printed out?

Details

A couple of summary statistics can be generated for emulators, based on their prediction errors on a validation set. This function produces the test statistic for a comparison to a relevant chi-squared distribution, and the similar test statistic for an F-distribution. In both cases, the expectation and standard deviation of the underlying distribution are also provided.

The output of this function is a logical vector stating whether the derived value lies within 3-sigma of the expected value. In systems where errors are expected to be correlated, higher weight should be given to the Mahalanobis measure than the chi-squared measure. Any anomalous results can be investigated in more depth using the individual_errors function.

validation_diagnostics 67

Value

Whether the observed value lies within 3-sigma of the expected value.

References

```
Bastos & O'Hagan (2009) <doi:10.1198/TECH.2009.08019>
```

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), residual_diag(), standard_errors(), validation_diagnostics()
```

Examples

```
summary_diag(SIREmulators$ems$nR, SIRSample$validation)
```

```
validation_diagnostics
```

Emulator Diagnostics

Description

Performs the standard set of validation diagnostics on emulators.

Usage

```
validation_diagnostics(
  emulators,
  targets = NULL,
  validation = NULL,
  which_diag = c("cd", "ce", "se"),
  analyze = TRUE,
  diagnose = "expectation",
  ...
)
```

Arguments

emulators	A list of Emulator objects.
targets	The list of observations for the outputs
validation	The validation set, containing all inputs and outputs.
which_diag	Which diagnostics should be performed (see description)
analyze	Should plotting and/or failing points be returned?
diagnose	For bimodal systems, should the expectation or variance be considered?
	Any additional parameters to pass to the diagnostics (eg sd, cutoff,)

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Details

All the diagnostics here can be performed with or without a validation (or 'holdout') set of data. The presence of a set of targets is optional for some checks but mandatory for others: the appropriate warnings will be given in the event that some checks cannot be applied.

The current options for diagnostics (with the codes for which_diag) are:

Standardised Errors (se)

Comparison Diagnostics (cd)

Classification Errors (ce)

All of the above (all)

For details of each of the tests, see the help file for get_diagnostic.

Value

A data frame containing points that failed one or more diagnostic tests.

See Also

```
Other diagnostic functions: analyze_diagnostic(), classification_diag(), comparison_diag(), get_diagnostic(), individual_errors(), residual_diag(), standard_errors(), summary_diag()
```

Examples

```
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, SIRSample$validation)
# data.frame of failed points (empty) and a 3x3 set of plots
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, SIRSample$validation,
    c('ce','cd'))
# empty data.frame and a 3x2 set of plots
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, SIRSample$validation,
    cutoff = 2, sd = 2)
# k-fold (with k = 3)
validation_diagnostics(SIREmulators$ems, SIREmulators$targets, k = 3)
```

validation_pairs

Validation Set Diagnostics and Implausibility

Description

Creates pairs plots on the set of validation points of diagnostic suitability and implausibility.

Usage

```
validation_pairs(ems, points, targets, ranges, nth = 1, cb = FALSE)
```

validation_pairs 69

Arguments

ems	The Emulator object(s).
points	The set of validation points to plot.
targets	The set of targets to match to.
ranges	If provided, this gives the plotting region (see above).
nth	The level of maximum implausibility to plot.
cb	Whether or not the colour scheme should be colourblind friendly.

Details

The plots are organised as follows:

- a) Emulated versus simulated output (lower diagonal). This is similar in spirit to comparison_diag: the plotted points are their location in the input space and the points are coloured by the emulator prediction's deviation from the simulator value.
- b) Implausibility (upper diagonal). The points are again plotted based on their location in input space, but their colouration is now based on the implausibility of the point.

If ranges is provided, then the plotting region is created relative to these ranges. This can be useful if on later waves of a history match and the plotting is to be done relative to the original input space, rather than the (reduced) parameter space upon which the emulators have been trained.

Value

A ggplot object.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), wave_dependencies(), wave_points(), wave_values()
```

Examples

```
validation_pairs(SIREmulators$ems, SIRSample$validation, SIREmulators$targets)
wider_ranges <- purrr::map(SIREmulators$ems[[1]]$ranges, ~.*c(-2, 2))
validation_pairs(SIREmulators$ems, SIRSample$validation,
    SIREmulators$targets, ranges = wider_ranges, cb = TRUE)</pre>
```

```
variance_emulator_from_data

Variance Emulator Creation (Deprecated)
```

Description

Trains hierarchical emulators to stochastic systems

Usage

```
variance_emulator_from_data(
  input_data,
  output_names,
  ranges,
  input_names = names(ranges),
  verbose = interactive(),
  na.rm = FALSE,
  ...
)
```

Arguments

```
input_data All model runs at all points.

output_names The observation names.

ranges A named list of parameter ranges

input_names The names of the parameters (if ranges is not provided).

verbose Should status updates be printed to console?

na.rm Should NA values be removed before training?

Optional parameters that can be passed to link{emulator_from_data}.
```

Details

This function is deprecated in favour of using emulator_from_data with argument emulator_type = "variance". See the associated help file.

For stochastic systems, one may emulate the variance as well as the function itself. This is particularly true if one expects the variance to be very different in different areas of the parameter space (for example, in an epidemic model). This function performs the requisite two-stage Bayes Linear update.

All observations are required (including replicates at points) - this function collects them into the required chunks and calculates the summary statistics as required.

All other parameters passed to this function are equivalent to those in emulators are the Bayes Linear adjusted forms.

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Value

A list of lists: one for the variance emulators and one for the function emulators.

References

Goldstein & Vernon (2016) in preparation

Examples

```
# Excessive runtime

# A simple example using the BirthDeath dataset

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v_{ems} = v_{emu} = v_{emu}

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```

wave_dependencies

Multiple Wave Inputs vs Outputs

Description

Given multiple waves of points, produce input-output plots for each pair.

Usage

```
wave_dependencies(
  waves,
  targets,
  output_names = names(targets),
  input_names = names(waves[[1]])[!names(waves[[1]]) %in% names(targets)],
  p_size = 1.5,
  l_wid = 1.5,
  normalize = FALSE,
  zero_in = TRUE,
  wave_numbers = ifelse(zero_in, 0, 1):(length(waves) - ifelse(zero_in, 1, 0)),
  ...
)
```

Arguments

waves The list of data.frame objects, one for each set of outputs at that wave.

targets The target values of the outputs.

output_names The outputs to plot, if not all are wanted. input_names The inputs to plot, if not all are wanted.

p_size Control for the point size on the plots: smaller is better for many plots.

1_wid Control for line width of superimposed targets.

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normalize If true, plotting is done with target bounds equal size.

zero_in Is a wave 0 included in the waves list?

wave_numbers Which waves to plot

... Optional parameters (not to be used directly)

Details

It can be useful to consider what the dependencies between the input values and output values are, to investigate the suitability of the chosen input ranges (i.e. if widening an input range could result in the targets being matchable). This function provides those plots.

For each output-input pair, a points plot is produced with the input value on the x-axis and the output value on the y-axis. The target bounds are superimposed as horizontal lines. The points themselves are coloured by which wave of history matching they came from.

These can show dependencies between specific outputs and inputs and, if points are clustering at the far left or right edge of a plot, can give an indication that the input ranges are unsuitable for matching the target.

Value

A grid of ggplot objects.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_points(), wave_values()
```

Examples

```
wave_dependencies(SIRMultiWaveData, SIREmulators$targets, 1_wid = 0.8, p_size = 0.8)
wave_dependencies(SIRMultiWaveData, SIREmulators$targets, c('nS', 'nI'), c('aIR', 'aSI'))

# For many plots, it may be helpful to manually modify the font size
wave_dependencies(SIRMultiWaveData, SIREmulators$targets) +
ggplot2::theme(text = ggplot2::element_text(size = 5))
```

wave_points

Multiple Wave Point Plotting

Description

Given multiple waves of points, produces pairs plots

wave_points 73

Usage

```
wave_points(
  waves,
  input_names,
  surround = FALSE,
  p_size = 1.5,
  zero_in = TRUE,
  wave_numbers = ifelse(zero_in, 0, 1):(length(waves) - ifelse(zero_in, 1, 0)),
  ...
)
```

Arguments

waves The list of data.frames, one for each set of points at that wave.

input_names The input names to be plotted.

surround If true, points are surrounded by black boundaries.

p_size The size of the points. Smaller values are better for high-dimensional spaces.

zero_in Is a wave 0 included in the waves list?

wave_numbers Which waves to plot

... Optional parameters (not to be used directly)

Details

Subsequent waves are overlaid on the same pairs plots, to determine the evolution of the non-implausible region. One-dimensional density plots are also created on the diagonal.

Value

A ggplot object

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_values()
```

Examples

```
wave_points(SIRMultiWaveData, c('aSI', 'aIR', 'aSR'))

wave_points(SIRMultiWaveData, c('aSI', 'aIR', 'aSR'), TRUE, 0.8)
# For many plots, it may be helpful to manually modify the font size
wave_points(SIRMultiWaveData, c('aSI', 'aIR', 'aSR')) +
ggplot2::theme(text = ggplot2::element_text(size = 5))
```

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wave_values

Multiple Wave Output Plotting

Description

Given multiple waves of points, produces pairs plots of the outputs.

Usage

```
wave_values(
  waves,
  targets,
  output_names = names(targets),
  ems = NULL,
  surround = FALSE,
  restrict = FALSE,
  p_size = 1.5,
  l_wid = 1.5,
  zero_in = TRUE,
  wave_numbers = ifelse(zero_in, 0, 1):(length(waves) - ifelse(zero_in, 0, 1)),
  which_wave = ifelse(zero_in, 0, 1),
  upper_scale = 1,
  ...
)
```

Arguments

waves The list of data.frames, one for each set of outputs at that wave.
targets The output targets.

targets The output targets.
output_names The outputs to plot.

ems If provided, plots the emulator expectations and 3-standard deviations.

surround As in wave_points.

restrict Should the plotting automatically restrict to failing target windows?

p_size As in wave_points.

1_wid The width of the lines that create the target boxes.

zero_in Is a wave 0 included in the waves list?

wave_numbers Which waves to plot.

which_wave Scaling for lower plots (see description)

Optional parameters (not to be used directly)

wave_values 75

Details

This function operates in a similar fashion to wave_points - the main difference is that the output values are plotted. Consequently, the set of targets is required to overlay the region of interest onto the plot.

To ensure that the wave numbers provided in the legend match, one should provide waves as a list of data.frames with the earliest wave at the start of the list.

The parameters which_wave and upper_scale control the level of 'zoom' on each of the lower-triangular and upper-triangular plots, respectively. For the lower plots, which_wave determines which of the provided waves is to be used to determine the output ranges to plot with respect to: generally, higher which_wave values result in a more zoomed-in plot. For the upper plots, upper_scale determines the plot window via a multiple of the target bounds: higher values result in a more zoomed-out plot. If not provided, these default to which_wave=0 (or 1 if no wave 0 is given) and upper_scale = 1. If the value provided to which_wave does not correspond to a provided wave (or one explicitly not included in wave_numbers), it defaults to the closest available wave to the value of which_wave.

If ems is provided, it should follow the same structure as waves: at the very least, it should contain all emulators trained over the course of the waves. The emulator predictions for a target are made by the emulator for that target whose ranges are the smallest such that contain the point.

Value

A ggplot object.

See Also

```
Other visualisation tools: behaviour_plot(), diagnostic_wrap(), effect_strength(), emulator_plot(), hit_by_wave(), output_plot(), plot_actives(), plot_lattice(), plot_wrap(), simulator_plot(), space_removed(), validation_pairs(), wave_dependencies(), wave_points()
```

Examples

```
wave_values(SIRMultiWaveData, SIREmulators$targets, surround = TRUE, p_size = 1)
wave_values(SIRMultiWaveData, SIREmulators$targets, c('nS', 'nI'), l_wid = 0.8)
wave_values(SIRMultiWaveData, SIREmulators$targets, l_wid = 0.8,
wave_numbers = c(0, 1, 3), which_wave = 2, upper_scale = 1.5)
# For many plots, it may be helpful to manually modify the font size
wave_values(SIRMultiWaveData, SIREmulators$targets) +
ggplot2::theme(text = ggplot2::element_text(size = 5))
```

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